

# Valuation and Risk Analysis of International Bonds

**Brian P. Murphy and David Won**

\*Reprinted with permission from The Handbook of Fixed Income Securities.

Brian P. Murphy, Product Manager, BARRA, Inc.

David Won, Consultant, BARRA, Inc.

## INTRODUCTION

The world of international fixed income can well be described as one composed of widely differing market structures governing an even more diverse set of assets. When addressing the problems of managing a portfolio of international fixed income securities, one must consider not only the specific nature of the bonds held, but also the institutions that govern the trading of the assets in and across the various markets. Further, one must be aware of the trading patterns extant in the markets themselves. So, for example, the investor considering the purchase of Japanese government bonds (JGBs) must understand the rules for withholding on coupon payments as well as the market norms for liquidity in this highly illiquid market.

Taken together, these issues present a substantial challenge for the international investor. These problems, however, are further complicated by the imprecise linkage between markets in the form of the freely floating<sup>1</sup> exchange rate system. Perhaps of greatest importance over much of the 80's and into the 90's has been the effect of currency movements on the returns earned by international investors. It is not enough to bet on those markets that exhibit the highest (local) return over one's investment horizon—one must also get the currency right. The answers to the questions of when and to what degree to hedge currency exposure are often the most critical determinants of the decision of an international investment strategy.

Before beginning the task of devising a global investment strategy, one must understand the elements of each market that play a significant role in defining the value of assets traded in each market. One cannot merely apply knowledge gained from investing in one's own domestic market. Simple extensions of the strategies that work well in one market may lead to extremely poor results in another.

Returning to the example of the JGB market, at the time of this writing cash JGBs are relatively illiquid. This stems from the fact that the Japanese fiscal year ends on March 31 and Japanese investors

covet riskless JGB assets for their year-end balance sheets. This combined with investor reluctance to lend bonds has led to a short squeeze on the 10-year cash JGBs<sup>2</sup>. Consensus forecasts are that the Japanese market will experience a correction after fiscal year-end.

Valuation analysis, therefore, has a decidedly market-specific component that, to be properly accounted for, requires substantial knowledge about each market individually. This knowledge includes not only a taxonomy of the types of assets traded in the market but also a full understanding of the institutions that regulate the market and the trading habits of the important classes of investors active in the market.

Turning to risk analysis, we find a similar set of issues. The traditional measures of risk in the context of a single market—duration and, more recently, convexity in both their standard and option-adjusted forms—lose all meaning in the context of an international portfolio. Although these statistics can be computed, the meaning of a 4.5 year duration for a portfolio of U.S. Treasuries, U.K. Gilts and JGBs is, at best, unclear. Before a duration number can be sensibly interpreted for a global portfolio, the markets represented in that portfolio would have to be assumed to undergo identical interest rate changes *simultaneously*. This assumption has not been valid for most markets over any recent period, and evidence to this effect is provided below.

Currency movements further compound both the magnitude and volatility of international fixed income asset returns. In fact, from a U.S. perspective currency fluctuations can account for up to two-thirds of the return and risk of an unhedged international fixed income portfolio. Within the recent past this has been most notable in the weaker members of the European Community where devaluations have left investors following high-yield strategies severely bruised. Currency risks can be hedged, but this too entails a cost. Therefore, in developing a strategy for investing internationally, managers must give adequate consideration to the currency aspects of their portfolio.

The purpose of this chapter is to describe, in the context of today's marketplace, approaches for addressing these difficulties. In the next section, we offer an approach for devising single-market valuation models to assist the investor in making trading decisions, marking portfolios to market, and so on. The approach described has been applied, with uniformly excellent results, to markets as diverse as the U.K. Gilt, U.S., Canadian, and Japanese investment-grade and

Eurocurrency bond markets. Having developed a framework for accurate valuation of fixed-income securities, we then turn to the issue of forecasting the risk of a portfolio of these securities. Again we first consider local market risk—those factors of risk relevant to an individual market—and then add the new dimension inherent in global portfolios, namely currency risk. Finally, we provide a brief discussion of an approach for addressing currency risk, which separates this risk from local market risk. This approach facilitates not only risk analysis but also the analysis of alternative investment strategies.

## **VALUATION ANALYSIS**

Clearly, the most important element in the value of a fixed-income security is the prevailing level of interest rates, because any fixed-income security may be thought of as a promised stream of future cash flows. These cash-flows are then discounted owing to the time value of money and the relative riskiness of the promise. Adjustments made for the relative riskiness of any individual cash flow stream are in general much smaller than the effects of the interest rate or term structure.<sup>4</sup>

Adopting this point of view, the first and most important task of valuation analysis is to accurately characterize the term structure. Elsewhere in this volume the determinants of the term structure for a given market are discussed. Briefly stated, these amount to the relative supply and demand for government securities, the market's view of the future prospects of the national economy, and the relative riskiness of those prospects. The term structure, therefore, is embodied in the prices of government securities, assuming that the sovereign borrower is the borrower of lowest risk in any fixed income market.

We can estimate the term structure at any point in time from the prices of government bonds. Although the estimation can involve weighting each asset in the estimation by relative liquidity, or amount outstanding, a very simple approach would assume all government securities are of equal interest to investors and that the only differentiating feature is the maturity of a given issue. Making this assumption, one can derive the term structure from a cross section of prices spanning the maturity spectrum of the government market.<sup>5</sup> We can now apply this set of interest rates to compute fair prices of other bonds not used in the estimation. This approach provides a reasonably good approximation of the value of any bond issued in the market so long as the issue is not "too different" from the government bonds

used in estimating the term structure. But what does "too different" mean?

A precise answer to this question depends crucially on the state of the market when the question is posed. We can generally improve the approximation provided by the term structure by identifying and valuing those asset features that distinguish one bond from another in the minds of investors. Obviously the level of credit risk, most often associated with the rating of an issue, is important for an investor's valuation of a bond. For corporate debt, the economic sector in which the firm operates provides information as to the relative riskiness of the firm's prospects. Option features, whether for exercise by the issuer or the purchaser, effect the relative valuation of otherwise similar instruments. In general, any asset feature or market condition (such as relative liquidity, tax treatment, market volatility in the case of options, etc.) that substantially changes investors' willingness to pay for an issue should be considered.

Identifying the differentiating features for a particular market is the key step in refining the valuation analysis that begins with the term structure. For the U.S. market, items of relevance to investors would include those mentioned above rating, sector, liquidity, option features—as well as relative coupon, degree of subordination and, the existence of a sinking fund, among others. The final step in valuation analysis is to take the term structure as determined from the prices of government bonds and estimate the value of exposure to the differentiating features. Having done this, one can compute the fair value for any issue in the market and compare that figure with market prices for rich/cheap trading opportunities, marking a portfolio of untraded assets to market or assessing a bid/offer on an asset not recently traded.

To formalize our arguments, we offer the following valuation model specification<sup>6</sup>.

(1)

$$PM_n(t) = \sum_T \frac{cf_n(T) * PDB(t,T)}{\exp[k_n(t) * (T - t)]} + \epsilon_n(t)$$

$$(2) \quad = PF_n(t) + \epsilon_n(t)$$

with

$$(3) \quad k_n(t) = \sum_j x_{n,j} * s_j(t)$$

where

$PM_n(t)$  = bond  $n$  market price at time  $t$

$PF_n(t)$  = bond  $n$  fitted price at time  $t$

$cf_n(T)$  = expected cash flow at time  $T$

$PDB(t,T)$  = discount for horizon date  $T$

$x_{n,j}$  = bond  $n$  exposure to factor  $j$

$s_j(t)$  = market's assessment of factor  $j$

$\epsilon_n(t)$  = bond  $n$  price error at time  $t$

$k_n(t)$  = bond  $n$  total yield spread at time  $t$

The price of each bond is given by the discounted stream of promised future cash flows. The total discount function for any particular bond consists of two components; the base line term structure given by  $PDB(t,T)$  and the bond-specific yield adjustment given by equation (3). The term structure  $PDB(t,T)$  is estimated from the prices of government bonds. This discount function represents the borrowing cost of the least risky borrower in the market-by assumption, the central government. The bond-specific component of the discount function describes how the cash flows of any individual

issue are differentially discounted owing to the specific features of the asset. Hence the valuation of any security is a function of the market-wide properties of the term structure  $\{PDB(t,T)\}$  and the values of the various distinguishing asset attributes ( $s_j$ ) together with an asset's promised stream of cash flows  $\{CF_n(T)\}$  and its exposures ( $X_{n,j}$ ) to these distinguishing attributes.

Before turning to a discussion of the differences among the world's bond markets, recall our assumption that the government issues on which the term structure is based are identical up to maturity in the eyes of investors. In the real world, however, this can be far from the truth, depending on the market. In the U.S. Treasury market issues have coupons ranging from 5 to over 15 percent, bills are issued as discounts; certain issues are subject to differential tax treatment (e.g., flower bonds); others can be stripped; and some recently issued securities are considered "on-the-run," a term connoting their very high liquidity. Therefore, a more careful analysis of the U.S. term structure should account for these differences among government issues in order to better define the true term structure of interest rates in the economy.

Similar considerations apply in most other government bond markets. Perhaps the most extreme example is again the Japanese market. The benchmark bond, currently the # 145 JGB issue, accounts for a huge share of daily trading volume. In fact, when the # 89 held benchmark status (late 1986 through early 1987), it occasionally accounted for over 90% of daily volume! This pattern has relaxed recently, but it does highlight the extreme nature of some of the world's bond markets. Exhibit 1 below provides a comparison of the relative liquidity observed in the five largest government bond markets by capitalization as of January 1993.

### Exhibit 1

Relative Liquidity of Government Bond Markets (January 1993)

Market	Number Traded	Actively Traded	Benchmark
U.S.	134	12	6
Japan	15	5	1
Germany	24	24	2
France	8	13	4
U.K.	8	12	9

Source: J.P. Morgan Securities, Inc.

Aside from the differences in liquidity among government issues, specific attributes of each bond can lead to significant differences in prices. For example, the relative coupon of an issue may cause the bond to be priced relatively rich compared to what would be predicted from the estimated term structure. Borrowing again from the Japanese experience, JGBs with coupons in the 7 to 8 percent range are rarely traded because investors covet the cash flow and cannot replace it with recently issued bonds carrying 5 to 6 percent coupons<sup>7</sup>. Many instruments have option features—calls, puts, extensions, conversions—which affect asset values<sup>8</sup>. Others have sinking funds (some with options) or have no maturity such as the War Loans of the U.K. Gilt market. Each of these attributes alters the value of the issue for the investor.

In terms of the model specified above, this implies that a set of attributes to which the market imputes value are associated with government bonds. For proper estimation of the term structure  $\{PDB(t,T)\}$  one must simultaneously estimate the values ( $s_j$ ) of these distinguishing features ( $x_{n,j}$ ). Only then will the true base line interest rates be revealed.

Our research has shown that the above specification is robust, both across markets and through time. In analyzing data from 13 government bond markets<sup>9</sup>, we found that by carefully specifying the set of attributes important in each market, bond prices can be accurately predicted. Exhibit 2 provides representative statistics showing the accuracy of the model in all 13 markets at two different points in time. The results clearly show that on average the local market valuation models are quite consistent across time.

## Exhibit 2

### Root Mean Square Error (RMSE) of Pricing Errors

Market	December 1991		December 1992	
	No. of Issues	RMSE	No. of Issues	RMSE
Australia	15	0.16	15	0.08
Belgium	14	0.32	16	0.50
Canada	43	0.33	38	0.37
Denmark	9	0.05	10	0.07
France	24	0.09	25	0.18
Germany	64	0.22	51	0.17
Italy	17	0.08	19	0.54
Japan	28	0.18	21	0.08
Netherlands	31	0.12	27	0.15
Spain	8	0.17	8	0.20
Sweden	7	0.11	6	0.01
U.K.	25	0.21	28	0.33
U.S.	140	0.19	140	0.21

Source: BARRA

Given this framework for valuing individual assets, one can apply the models to such tasks as evaluating trading opportunities, marking a portfolio to market, or any other valuation-dependent analysis. Of course, there are caveats which apply in properly interpreting the results. The parameters estimated for each market are only as good as the data used in estimating them. Ideally, all prices used would reflect transactions occurring at nearly the same time. This is rarely, if ever, the case with bond price data because the source of the price is often a trader asked to quote a fair market value rather than a report of an actual transaction price. In light of the aforementioned variations in liquidity within each market, it may well be the case that only a very small fraction of the prices are "real" in this sense. One may then view portfolio valuation analyses as reasonable since the errors will tend to cancel each other when the underlying models are correct on average.

With valuation of fixed-income securities now well defined, we turn to the assessment of risk in fixed-income markets. The next section explores the issues involved in modeling risk for single-market

portfolios as well as multimarket portfolios which face the additional dimension of currency risk.

## **RISK ANALYSIS**

In the preceding section we argued that interest rates are by far the most important determinant of asset value in fixed income markets. Following directly from this, we argue that *interest-rate risk* is the most important element of risk. Our research shows that interest rate risk is roughly 90% of the story in most markets. We can improve on this; that is, we can explain even more market risk, by understanding the risk in the other factors of asset value that we identified in developing our valuation models. By modeling the linkages among the interest-rate and value factors, our risk characterization is further refined.

The remainder of this section explains our approach to modeling risk in a single fixed income market. Having devised risk models for the separate markets in a similar fashion, we then describe a model that accounts for the linkages existing across markets—essentially the covariance structure of market returns. This model considers the covariation of local market returns. The currency issue is addressed briefly in the next section.

### **Local Market Risk**

The variation in the term structure of interest rates accounts for the greatest amount of risk in a fixed income market. In fact, in the case of the U.S. bond market, interest-rate fluctuations account for roughly 90 percent of the risk. The remaining 10 percent is attributable to the risks inherent in asset features such as call or put options, sector membership, relative coupon, and rating. Exhibit 3 shows the decomposition of return variance for a universe consisting of all U.S. corporate bonds with a minimum maturity of one year as of January 1, 1993. The exhibit provides evidence in support of the claim that interest rates are the dominant element of risk. The fact that interest rates are the dominant risk factor explains the usefulness of duration as a good first approximation to bond risk. Duration measures risk relative to *small* (parallel) shifts in interest rates.

### Exhibit 3

#### Decomposition of Return Variance

(BARRA All Corporate Index - January 1, 1993)

---

	<u>Variance %2</u>
Interest Rate Effects	35.09
Interest Sensitive Factors	2.08
Covariance of Interest Rate Effects and Interest Sensitive Factors	-0.14
Quality and Sector Factors	5.39
Covariance of Quality and Sector Factors, and Interest Rate Effects	-3.59
Specific Issues	0.01
TOTAL	38.84

Source: BARRA

---

But, interest rates do not always shift in parallel<sup>10</sup> and the shifts are not necessarily small over time periods even as short as a week. Also, the extent to which the other valuation factors shift in unison with or in opposition to the term structure can markedly effect the risk of individual securities. Portfolios more highly exposed to these factors than the market as a whole, even while being duration matched versus a benchmark, can exhibit substantially different returns than the market index portfolio.

How then does one improve upon the duration approximation of fixed income risk? Since the risk of an asset is defined as the volatility of the asset's returns and the returns are driven by uncertain changes in interest rates and spreads, it is natural to relate fixed income asset risk to term structure movements; or more generally to the valuation models described above. After the valuation model has been specified, valuation factor returns can be computed over the investment horizon of interest, say one month. The result from this exercise is a set of factor returns series embodying the returns experienced within the local market over the history used in the analysis. These factor returns

can then be used in constructing a covariance matrix that reveals how the various local market factors move with respect to one another.

The preceding paragraph contains the generalized "recipe," if you will, for a full multiple factor risk model for a fixed income market. By mapping an asset's, or portfolio's, factor exposures into the covariance matrix it is possible to estimate the risk of an asset or portfolio. The key element in the risk model is the specification of the valuation model described in the previous section. The extent to which this underlying valuation model captures the important factors for explaining value in the market will set the limits on the risk model's ability to explain variation in market related returns. Further, by modeling asset specific risk, it is possible to expand the capability of the model to explain variation in total asset and portfolio risk.

Recalling our general valuation model, the factors explaining value in a market are the term structure (equivalently, the prices of pure discount bonds at various maturities) and the features of assets that differentiate otherwise equivalent bonds. The valuation model produces a price for each of these distinguishing features along with the pure discount bond prices.

The number of factors in any particular model will depend on the variety of assets found in the market being modeled. Taking the U.S. Treasury market as an example, the interest-rate structure can be defined very accurately by fewer than 10 interest rates. The special features in this market contributing to differential asset prices number two or three.<sup>11</sup> Therefore, our risk model has roughly a dozen factor returns series from which the covariance matrix is estimated.

So far this example has focused on the U.S. fixed income market—in fact, only the Treasury market. However, through suitable generalization of the underlying valuation model, this approach can be applied in any fixed income market. To date, the valuation methodology has been applied to the U.S., Canadian, and Japanese investment-grade markets, the U.K. Gilt market, the Australian government and semi-government market and the Eurocurrency markets—all with very strong performance.

In addition, the risk modeling approach has been applied to all of these save the Eurocurrency markets,<sup>12</sup> as well as 13 government markets covered by the J. P. Morgan Government Bond Indexes and the Salomon Brothers World Government Bond Indexes. For each of these 13 markets, the risk models using between 9 and 14 parameters

have performed remarkably well, explaining between 85 and 95 percent of risk.

### **Global Portfolio Risk**

We began this chapter with the purpose of describing methods for valuing and assessing the riskiness of international bonds and portfolios. So far we have described techniques for valuing and assessing risk of issues in a single-market context. Here we address the complication of risk analysis for portfolios composed of bonds from several markets. Again, we postpone the assessment of the currency factor until the next section.

To this point, we have devised individual market models. Each of these models, both valuation and risk, is based on roughly 10 valuation parameters. Before proceeding with the development of a multimarket risk analysis framework, we offer in Exhibit 4 evidence of the inappropriateness of duration as a measure of risk for a multimarket portfolio.

The exhibit shows the correlation of returns across markets over a period of roughly 5 years. For duration to make sense in the context of portfolios of bonds from several markets, the correlations would have to be identical, barring differences in duration across markets. In other words, if we control for the differences in duration between individual markets, then the return correlations would be unity. While individual market durations are different and vary somewhat through the sample period of January 1988 through December 1992, this does not explain the range of correlations. These are shown to range from a low of 0.076 to a high of 0.950. Clearly then, duration is a poor approximation for risk in the context of multicurrency fixed income portfolios.

We can produce a more general risk analysis framework by using the risk models for the component markets. However, to move to such a multimarket framework, we must overcome one important difficulty. In the case of each single-market model, we suggested computing a covariance matrix from roughly 10 parameters. This entails estimating approximately 55 elements for each market's covariance matrix. However, simply combining the return series and attempting to estimate the resulting multimarket covariance matrix would require enough data to reliably estimate over 8500 parameters! Some alternative approach must be used.

**Exhibit 59-4  
Correlation of Fixed Income Returns (January 1988 - December 1992)**

	Australia	Belgium	Canada	Denmark	France	Germany	Italy	Japan	Netherlands	Spain	Sweden	U.K.	U.S.
Australia	5.533%	0.262	0.327	0.165	0.218	0.282	0.076	0.202	0.314	0.208	0.175	0.294	0.097
Belgium		3.608%	0.178	0.816	0.858	0.930	0.615	0.487	0.950	0.678	0.516	0.603	0.335
Canada			5.828%	0.209	0.158	0.188	0.155	0.213	0.154	0.212	0.209	0.296	0.493
Denmark				4.201%	0.794	0.804	0.638	0.403	0.810	0.692	0.558	0.584	0.268
France					4.466%	0.867	0.648	0.463	0.861	0.669	0.497	0.587	0.336
Germany						3.646%	0.599	0.550	0.945	0.638	0.513	0.623	0.357
Italy							2.920%	0.329	0.641	0.655	0.453	0.516	0.153
Japan								4.195%	0.533	0.329	0.280	0.492	0.292
Netherlands									3.363%	0.694	0.509	0.606	0.322
Spain										3.362%	0.428	0.543	0.137
Sweden											4.950%	0.420	0.234
U.K.												6.241%	0.255
U.S.													4.091%

Note: Diagonal elements represent annualized standard deviation of returns.

\*Source: BARRA

Research at BARRA has identified several methods to reduce the number of factors in the covariance matrix. The first approach, discussed in an earlier version of this article, used the technique of principal components analysis to reduce the dimensionality of the problem. Over the past five years BARRA has developed a new methodology to estimate the covariance matrix using a returns-based analysis approach. This method, which can be viewed as an extension of the principal components approach, creates a covariance matrix using actual bond returns thereby better capturing the risk inherent in each local market.

From earlier work, we learned that the total risk of a bond can be significantly explained using three factors. Studying the results of principal components analysis suggests the movements that best explain term structure dynamics in each local market. We describe these generalized movements as Shift, Twist and Butterfly motions. Exhibit 5 presents the one standard deviation Shift, Twist and Butterfly movements, in spot rate space, defined within BARRA's global fixed income model for the United States government bond market.

As mentioned above the shift, twist and butterfly shapes are the "most likely" movements the local term structure will undergo in reshaping. Conceptually each shape and its inverse should be overlaid on top of the current spot rate curve to provide a two-standard deviation probability for movements of the spot rate curve over the coming year.

The *shift* factor, which has the highest explanatory power of all three factors for each local market, is defined as the generalized motion whereby all rates move in the same direction. The shift for most countries is downward sloping with maturity indicating a higher volatility for shorter term rates. For example, in the United States, a one standard deviation shift in the term structure will increase short rates (1 yr.) by approximately 180 basis points and long rates (30 yr.) by approximately 100 basis points. Conceptually this implies that in the current interest rate environment with short spot rates at roughly 3.75% and long spot rates at approximately 7.00%, in 2 out of 3 years we would expect the short rates to be within 2.85% and 4.65% and the long rates to be within 6.50% and 7.50%.

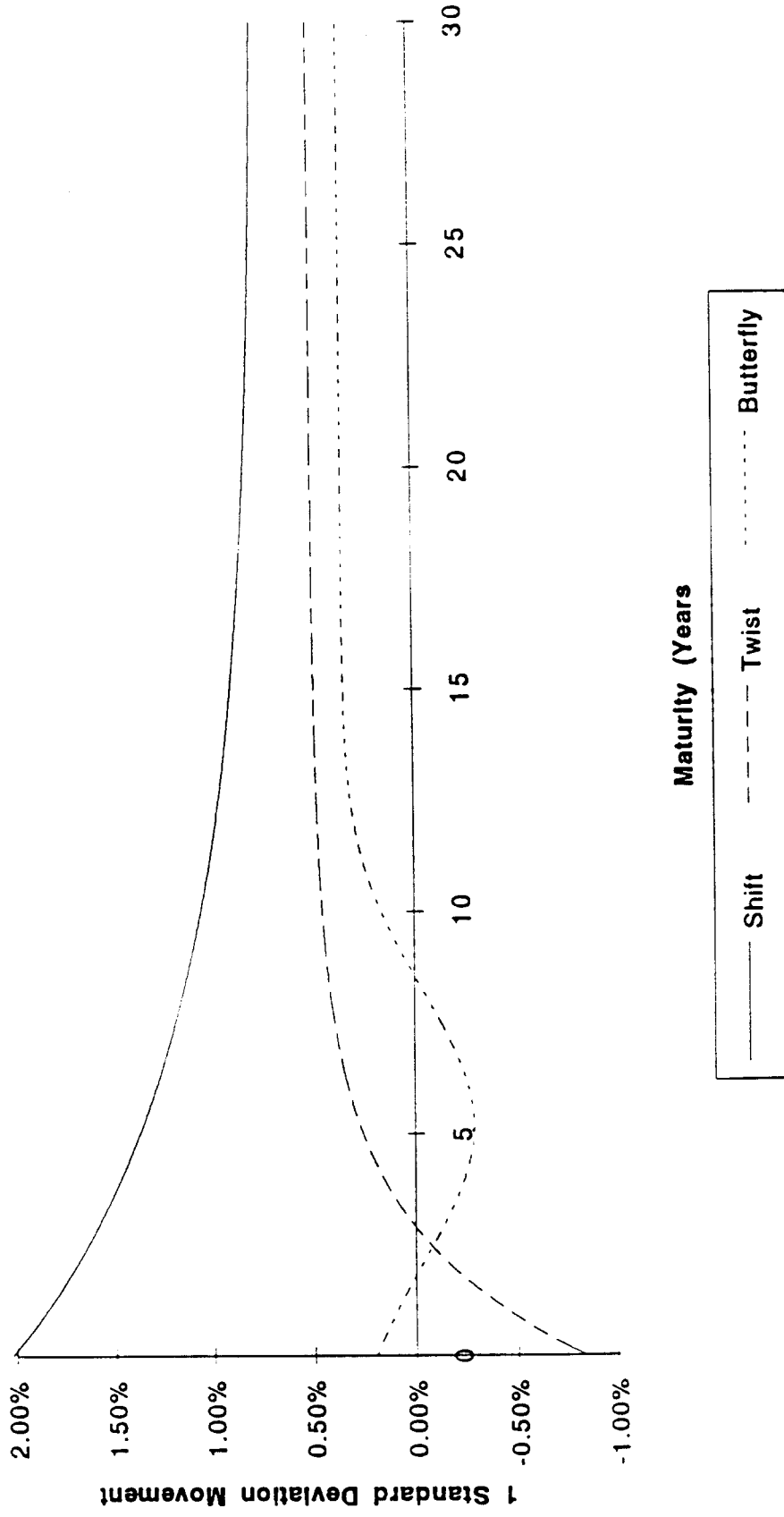
An interesting result occurs if one constrains the shift factor to be a parallel shift of 100 basis points across the spot curve. In this case the shift factor becomes duration; or the market's sensitivity to small

parallel movements in the underlying spot rate term structure. Therefore, because the shift factor is a more generalized form of duration, it should explain more local market risk than duration.

The second explanatory factor is a *twist* in the term structure. In this case, we see short and long rates moving in opposite directions. As we can see from the exhibit, a one standard deviation twist in the United States will decrease short rates by 80 basis points and increase long rates by 50 basis points. The third factor can be interpreted as a *butterfly*, whereby short and long rates move in the opposite direction to medium rates. In the United States, the average increase in short and long rates is approximately 30 basis points with medium rates falling by 20 basis points.

(INSERT Exhibit 5 Excel 3.0 USA Annualized Spot Rate Movement)

# USA Annualized Spot Rate Movement



Once the factors have been defined on a country by country basis they are held constant going forward until the model is re-estimated due to changes in the markets covered. The monthly returns of all bonds can then be attributed to the Shift, Twist and Butterfly factors by running the following regression for each country:

(4)

$$r(n,t) = x_s(n,t) \cdot r_s(t) + x_t(n,t) \cdot r_t(t) + x_b(n,t) \cdot r_b(t) + \mu(n,t)$$

where

$r$  = excess return to bond  $n$

$x_s$  = exposure of bond  $n$  to shift movement

$x_t$  = exposure of bond  $n$  to twist movement

$x_b$  = exposure of bond  $n$  to butterfly movement

$r_s$  = return to shift component

$r_t$  = return to twist component

$r_b$  = return to butterfly component

$\mu$  = residual return

Exhibit 6 presents a comparison of Shift, Twist, Butterfly (STB) analysis with traditional Duration and Duration/Convexity analyses. These results indicate that the STB approach offers advantages over duration and duration/convexity in terms of predictive power. Additionally, because the STB factors are defined shapes this framework is particularly well suited to performance attribution and analysis.

## Exhibit 6

### Risk Explained by Various Modelling Techniques

(Weekly data: January 1990-June 1991)

---

Market	Duration	Duration Convexity	Shift/Twist Butterfly
Australia	70.77	86.14	86.56
Belgium	61.45	65.06	69.09
Canada	76.00	85.87	89.97
Denmark*	NA	86.21	93.16
France	69.11	77.13	79.53
Germany	70.98	77.39	77.58
Italy	56.56	65.49	68.66
Japan	72.99	78.12	81.34
Netherlands	79.85	85.03	84.84
Spain	41.95	45.13	55.99
Sweden*	NA	NA	96.53
UK	75.70	83.45	86.22
USA	79.72	89.22	92.80

---

\* Denmark and Sweden use monthly data over same time period.

Source: BARRA

---

Thus, our analysis of the covariance matrix shows at most three risk factors in each of the single-market models. These 39 factors (three per market, 13 markets) can now be used to construct a risk model allowing for the analysis of portfolios with assets from any market. Of course, the currency issue remains and we now turn to this aspect of the analysis of international portfolios.

### Currency Risk

A full analysis of the currency aspect of international investment is a topic for a book in itself. However, we can say a few things regarding currency in the context of the multimarket risk model we have developed in this discussion. Thus far we have a risk model that captures the "within market" variation of returns as well as the relationships of the shift, twist, and butterfly factors across markets. Further, the analysis shows that the use of shift, twist and butterfly factors sacrifices nothing in terms of the power of the model to

account for risk. Now, if exchange rates were perfectly determined by interest rates, the story would be basically finished. However, other factors enter into the equation of exchange rate determination, so currencies must be added explicitly.

The final step in constructing our risk model for multicurrency portfolios is adding currency returns to the covariance matrix. It is important to realize that we must carefully define the currency returns to ensure that the separation between local market returns and currency returns is valid.

In deriving the separation property of currencies versus securities, we require a few definitions:

$rx$   $\equiv$  return due to exchange rate changes

$rc$   $\equiv$  exchange return plus short-term interest (currency return)

$rfl$   $\equiv$  local risk-free interest rate

From these definitions, we have the following relationship:

$$1 + rc = (1 + rx) \times (1 + rfl)$$

$$(5) \quad rc = rx + rfl + (rx \times rfl) \approx rx + rfl$$

This says that an investment in a currency earns not only the return from changes in the exchange rate with the base or numeraire currency but also the risk-free rate in the foreign market. For example, a U.K. investor buying dollars would earn the return for changes in the £/\$ rate plus the interest on the short term instrument, in this case a Treasury bill.

Now we turn to the returns to assets or bonds in the fixed income case. Again we require a few definitions:

$rl$  = total return on an asset for the *local* investor

$rfn$  = risk-free return in the numeraire market

$r$  = asset return from the numeraire perspective

Thus,

$$(1 + r) = (1 + rx) \times (1 + rl)$$

$$(6) \ r = rx + rl + (rx \times rl) \approx rx + rl$$

Finally we translate this numeraire total return figure to numeraire *excess* return:

$$(7) \ \text{Numeraire excess return} \equiv (1 + r) - (1 + rfn) = r - rfn$$

From these definitions, we can use a simple algebraic manipulation to derive the separation property we are seeking:

$$r - rfn = rx + rl + (rx \times rl) - rfn$$

$$(8) \ r - rfn \approx (rx + rfl - rfn) \quad + (rl-rfl) \\ = \text{currency excess} \quad + \text{local excess}$$

Interpreting equation (8) we see that from the numeraire perspective, that is the investor's domicile, excess return to an asset is approximately<sup>13</sup> equal to the sum of the numeraire excess currency return and the local excess return of the asset. We now have the desired separation of currency and local market returns.

Although the above exercise is required to complete the risk model, one distinct advantage of separating the currency and local market return components within our multicurrency risk model is that we can estimate each portion separately. Research at BARRA has concluded that currencies tend to trade much differently<sup>14</sup> than fixed income assets and that it is possible to estimate a much more robust "currency block" by exponentially smoothing the currency return data.

By defining returns in the manner described above, the full risk model is complete and one can analyze the risk of any portfolio of bonds and currencies.

## **SUMMARY**

This chapter has presented approaches first for constructing valuation models for fixed-income securities and second for assessing the risk in portfolios of these assets. The linkages across markets in the form of currencies have also been addressed in a manner which facilitates the analysis of risk by separating the local market aspects from those purely attributable to currency fluctuations.

The valuation models proposed are based on the estimation of interest rates, or term structures, in each market along with the market-specific factors that distinguish assets from one another. Interest rates are found to be of greatest importance in valuing securities. However, by identifying features of assets, in addition to interest rates that investors view as important, we are able to refine further the valuation models.

Having developed for each market a valuation model based on about ten or twelve parameters, we have shown that the risk in these markets can be explained by only two or three parameters. Using the technique of Shift, Twist and Butterfly analysis, we derived a set of hybrid parameters that were relatively independent within each market. We showed that shift, twist, and butterfly factors captured essentially all of the explanatory power of the original dozen or so parameters. Hence we were able to reduce the dimensionality of the overall model to a manageable number.

Finally, we showed that by suitably defining returns, we could separate the local market returns from currency returns. This facilitated the analysis by making local market return independent of the investor's domicile and added to the robustness of the model by allowing us to estimate the currency and local market blocks of the covariance matrix separately.

<sup>1</sup>Throughout the 1980's there were tight linkages among the European markets imposed by the EMS. However September 1992 showed that artificially pegging exchange rates of countries at various stages of their economic cycles can be hazardous to central banks wealth! The Dutch and German markets provide an interesting example where policy has long been targeted to the maintenance of a preset exchange rate with monetary policy adjusted to keep currencies in line.

<sup>2</sup>The bulk of large capitalization JGB coupon issues are 10-year issues. There are 2-, 3-, and 4-year coupon bonds along with 5-year discount issues in the JGB market, however there is virtually no secondary market. For this reason, the major indexes exclude these issues from their constituent lists. There are also a few issues with 20-year maturities. These generally are included in indexes representing the JGB market.

<sup>3</sup>An insightful discussion of this can be found in "Fixed Income Risk Modeling" by Ronald N. Kahn, also in this volume.

<sup>4</sup>The exception to this is the very risky class of bonds known as high yield, or junk, bonds. Spreads over the term structure are normally a substantial proportion of the underlying Treasury rate and can approach the level of the term structure during periods of heightened economic uncertainty.

<sup>5</sup>Again, technical details related to this estimation are discussed elsewhere in this volume as well as in numerous academic articles.

<sup>6</sup>This analysis is derived from that in "Fixed Income Risk Modeling" by Kahn. See footnote 4.

<sup>7</sup>The current benchmark bond, the #145, has a coupon of 5.5% while other more recent issues have coupons in the range of 6.0%.

<sup>8</sup>For example, as of the beginning of 1993 approximately 15% of the Italian government's liquid long-term debt were puttable instruments.

<sup>9</sup>The data for this research was provided by Salomon Brothers, Inc. and J.P. Morgan Securities, Inc. The result of the research efforts are PC-based computer systems for assessing the risk of portfolios of government bonds from the represented markets—the BARRA Global Bond System.

<sup>10</sup>Quite the contrary. A study performed by BARRA showed that for weekly intervals from 1979 through 1986 the term structure as estimated by BARRA never once shifted in a perfectly parallel fashion and often moved in a rotating or butterfly-like fashion.

<sup>11</sup>These would include coupon differential, on-the-run status, and tax treatment.

<sup>12</sup>The limitation here is the lack of a reasonable history of reliable pricing information.

<sup>13</sup>For this approximation, we are ignoring the cross product term  $r_x * r_l$  which, for all practical purposes is very close to zero over one-month horizons. Note, however, that we do not ignore the correlation of  $r_x$  and  $r_l$ , which is important for risk analysis.

<sup>14</sup>Research conducted by BARRA has shown that as the time frame gets shorter, currency returns become more non-normally distributed. In short, currencies trend.