OAS Analysis for CMOs

Essential to the understanding of CMO behavior.

Oren Cheyette

OREN CHEYETTE is senior vice president of Capital Management Sciences in Los Angeles (CA 90025).

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It is customary to buy and sell investment-grade fixed-income securities on the basis of yield quotation, usually expressed as a spread over some maturity point on the Treasury curve. This is convenient as a trading convention for two reasons.

First, it is simple, in that one needs only a cash flow projection to go between yield and price. It doesn’t matter where the cash flow projection comes from, so long as buyer and seller agree on it.

Second, because a primary determinant of bond yields is the level of the Treasury market, trading on the basis of yield spread captures the main short-term variation in value. Except for securities whose value is dominated by embedded options, yield is a reasonable rough approximation to expected return.

While this pricing method simplifies trading, few investment professionals would treat the nominal yield spread as a precise measure of relative expected return. Yet many investors continue to use nominal spread as a “rich/cheap” index to determine the relative attractiveness of presumably similar securities.

For example, stable intermediate PACs on par collateral might trade 50 to 80 bp “off the curve”; broken PACs with premium collateral might carry an additional discount of 50 to 100 bp; support tranches would trade wider still. Investors persist in attempting to adjust these nominal spreads for prepayment risk in order both to compare a security to other similar ones and to compare the different sectors. These comparisons are really “rule of thumb” exercises, partly for lack of familiarity with, availability of, or inconvenience of the quantitative analytical tools required for accurate analysis.

The standard method for incorporating the present value of future cash flow variation into the valuation of a fixed-income security is
known as option adjusted spread (OAS) analysis. OAS analysis is the application of discounting to the case of uncertain future cash flows. While it has its grounding in basic finance, the method has not gained widespread acceptance in the mortgage-backed security market, even though the value of many CMO tranches is substantially affected by their option component, i.e., their cash flows are extremely variable as a function of interest rates. This is so for IOs, POs, inverse floaters, most support tranches, TACs, and subordinate and broken PACs.

There are a number of possible reasons OAS analysis is not used. One is the “black box” character of OAS calculations. Investors do not generally create their own OAS calculations. Rather, they rely on dealer firm calculations in printed reports or on on-line software whose inner workings are a partial or complete mystery.

An associated problem is that OAS estimates even for generic collateral show wide variation among providers. These variations are attributable to differing interest rate and prepayment modeling assumptions that can be hard to determine and understand.

It is certainly true that the analytical tools required for OAS analysis require more effort to understand than those used in yield calculations. There are a number of modeling issues that must be resolved in the course of specifying an OAS algorithm. They include the number of factors governing term structure fluctuations, and the interest rate and maturity dependence of the volatility, as well as the nature of the numerical algorithm that implements the model.

A second difficulty is that MBS analysis requires a prepayment model, which is generally derived by a statistical fit to the parameters of a non-linear algorithm. Probably most variation among providers of OAS is because of differences in their prepayment models. Because the details of dealer prepayment models tend to be closely guarded, it is hard to argue that one can become comfortable with a model just by diligent study.

It is also not true that all models give similar results. Street models seem to differ significantly in the modeling of pool burnout, for example. Statistically fitted prepayment models also lag systematic changes to the market, such as the boom in low- or zero-cost refinancing that has helped to maintain the recent high level of prepayments and apparently shortened the seasoning period for new loans relative to historical experience.
The problem with avoiding OAS models for want of a perfect prepayment model is to find a replacement. A statistically derived prepayment model will always be slightly out of date, and probably will provide only a crude forecast for conditions, say, ten years out. But any attempt to measure value depends on having some means to estimate future cash flows. So the cruder measures of value commonly used, such as fixed PSA yield or vector analysis, are subject to the same drawbacks, while making less complete use of currently available information.

A third objection to use of OAS stems from an incomplete understanding of the Monte Carlo algorithms in general use for MBS valuation. The typical interest rate scenarios generated by a Monte Carlo algorithm appear to imply an expectation (of the model) that interest rates will, on average, tend to follow the forward curve. In the steep yield curve environment of the last few years, this implies drastically slowing prepayments, and, for example, makes IOs look extraordinarily cheap while POs and inverse floaters look rich. Seeing these results, practitioners can be inclined to dismiss OAS as a useful measure of value, especially because, in retrospect, holders of IOs have been burned, while those who owned POs or inverse floaters have done extremely well. (Except since the beginning of 1994!)

The “rising rates” problem has been addressed in an article by Asness [1993], who shows clearly the arbitrage basis of OAS models. This still leaves a puzzle: Why do investors appear to misprice these securities so badly compared to the predictions of OAS analysis? I will argue that in fact the IOs and POs are properly valued by OAS models and may at the same time be fairly priced by the market. The resolution to this paradox involves an examination of the risks to an investor in these derivative securities.

I first briefly review the basic principles of OAS analysis and discuss some subtleties in the definition of OAS. Readers interested only in the application of OAS to CMOs can skip this review.

**THEORY OF OAS ANALYSIS**

There are a number of ingredients required for OAS analysis. They include a present value formula for uncertain cash flows, an interest rate model, and a numerical algorithm. Following are brief discussions of these components.
Present Value

The procedure for valuation of a security with known cash flows is familiar to anyone who deals with fixed-income markets. For each future cash flow date, there is a spot interest rate that determines the discount factor for the cash flow; the present value of the security is the sum of the product of its cash flows and their discount factors.

This formula is compactly expressed as

\[
PV = \sum_{(\text{periods } n)} \frac{CF_n}{(1 + r(n))^n}
\] (1)

where \( r(n) \) is the spot rate to payment period \( n \), and \( CF_n \) is the cash flow received at period \( n \) (assuming equally spaced cash flows, and expressing interest rates in periodic rather than annualized terms). The formula can also be written as

\[
PV = \sum_{(\text{periods } n)} \frac{CF_n}{(1 + f_1)(1 + f_2)...(1 + f_n)}
\] (2)

where \( f_i \) (\( i = 1, ..., n \)) are the one-period forward rates between now and period \( n \).

The relationship between the first and second formulas can be taken as defining the forward rates, as one can use these formulas recursively starting from short-maturity bonds and working out the curve to derive the forward curve in terms of current spot rates:

\[
(1 + f_1)(1 + f_2)...(1 + f_n) = (1 + r(n))^n
\] (3)

If there were no uncertainty about future interest rates, it is evident that future spot rates would be given in terms of today’s forward rates. This relation holds because in the absence of uncertainty there is no risk in owning a security with known cash flows of whatever term. All securities’ one-period returns would therefore be equal to the return on a one-period bond.

This arbitrage relationship implies that future one-period spot rates would be equal to the current one-period forward rates (hence the name), i.e., denoting the future one-period spot rates by \( r_i \):

\[
(1 + f_1)(1 + f_2)...(1 + f_n) = (1 + r_1)(1 + r_2)...(1 + r_n)
\] (4)

In the absence of uncertainty, then, Equation (2) implies that the present value of a cash flow is equal to its future value discounted by
the cumulative future one-period rates. In this way we have expressed the present value in terms \textit{future} interest rates (which happen to be known with certainty today).

For known (fixed) cash flows in the presence of uncertainty (i.e., interest rate volatility) Equations (1) and (2) still hold. These are really definitional formulas for interest rates, so this is just saying that equal future cash flows have equal present values. Uncertain cash flows (i.e., cash flows dependent on future interest rates) are valued by a simple generalization of Equation (2).

The general valuation formula for uncertain cash flows is

\[
PV = \lim_{N \to \infty} \frac{1}{N} \sum_{\text{paths } r^*} \sum_{\text{periods } n} \left( \frac{\text{CF}_{n}^*}{(1 + r_1^*)(1 + r_2^*) \ldots (1 + r_n^*)} \right)
\]

where \( r^* \) denotes a \textit{risk-neutral} (RN) interest rate path, with \( r_i^* \) equal to the future one-period interest rate for period \( i \); \( \text{CF}_{n}^* \) is the cash flow to be received in this scenario at period \( n \); and the limit and first summation simply denote averaging over a large number of scenarios whose distribution models the interest rate dynamics.

Application of this equation to a security with known cash flows implies a constraint on the interest rate dynamics: The average over scenarios of the cumulative discount factors must be equal to the current discount factor to any period:

\[
\frac{1}{(1 + f_1)(1 + f_2) \ldots (1 + f_n)} = \lim_{N \to \infty} \frac{1}{N} \times \sum_{\text{paths } r^*} \left( \frac{1}{(1 + r_1^*)(1 + r_2^*) \ldots (1 + r_n^*)} \right)
\]

It is in this sense that the future risk-neutral spot rates \( r^* \) are said to “follow” the current forward curve. This formula is the generalization of Equation (4).

Equation (5) can be derived by an arbitrage argument analogous to that used in deriving the Black-Scholes option pricing formula for one asset, except that in this case we have multiple assets, namely, a different zero-coupon bond for each cash flow date, and constraints on their relative price movements. The basic idea is the same, though:
1. Each uncertain cash flow is equivalent (in terms of interest rate risk) to some amount of a zero-coupon bond maturing on the cash flow date.

2. The present value of the zero-coupon bonds is known in terms of the current yield curve.

3. In a risk-neutral world, the zero-coupon bond price is also expressible in terms of future interest rates through Equation (6).

Combining these three principles leads to the valuation in Equation (5). The simple reason why Equation (5) is expressed in terms of risk-neutral interest rates is just that it is possible and convenient to do so: the present value of uncertain cash flows is independent of the market’s risk aversion except as reflected in the prices of bonds with known cash flows.

The valuation formula can be re-expressed with risk-averse rate paths — i.e., the scenario probabilities can be determined by some estimate of market risk aversion — instead of risk-neutral ones. The risk-averse distribution of interest rate paths is centered around an average that lies below the forward curve, thus rewarding holders of longer and therefore riskier debt with an expected return higher than holders of cash.

In this case Equation (6) no longer holds: The average based on risk-averse probabilities of the cumulative one-period discount factors on the right-hand side is greater than the current spot discount factor on the left-hand side, reflecting the excess expected return that compensates for the risk. That is, the expected return on holding cash to some future date is less than the return on a zero-coupon bond maturing then.

The pathwise discounting formula that replaces Equation (6) is significantly complicated by the inclusion of the market price of risk, and the additional complication is ultimately pointless: The change in the distribution of paths (and the resulting cash flows) is exactly offset by the change in discount factors, giving the same numerical result as the risk-neutral formula. This is the significance of arbitrage pricing.

**Interest Rate Model**

The interest rate dynamics are governed by two arbitrage conditions. First, the risk-neutral short rate path distribution has to satisfy Equation (6). That is, the interest rate dynamics must reflect the current term structure.
Second, the evolution of the term structure as a whole is governed by a dynamic arbitrage condition that ensures that two equally risky portfolios have the same expected return. In the case of a model with parallel fluctuations of yields across maturities (e.g., the Ho-Lee model), this condition implies a gradual steepening of the yield curve, with a rate proportional to the cumulative variance of interest rates. Without mean reversion, this ultimately produces unreasonable yield curves, but in models with mean reversion it leads to fairly natural looking curves: steeper at the short end than the long end, steep when rates are low, and inverted when rates are high.

Subject to these constraints, the rate dynamics may be arbitrarily specified to match one’s outlook. There are a number of choices to be made. One is the number of factors modeled. A one-factor model treats yield fluctuations across the curve as perfectly correlated instantaneously. That is, given the change in short rate and knowledge of the volatility term structure, one knows the change in the long rate. (The long-rate change is less than the short-rate change if there is mean reversion.)

While not completely faithful to the observed dynamics, one-factor models give reasonable valuation of securities whose cash flows depend predominantly on the overall level of rates. Moreover, they do not require forecasts of many volatility parameters to determine the future dynamics. For example, the one-factor model used in CMS’s PACE system depends on two parameters: short-rate volatility and long-rate volatility (equivalently, short-rate volatility and mean reversion strength).

Multifactor models, while they more accurately model the real world, require more parameter estimates. A two-factor version of PACEs model requires four parameters: two short-rate volatilities and two long-rate volatilities. Some two-factor variants require five or more parameters.

A second issue is the dependence of volatility on interest rates. Standard choices are constant, linearly proportional (volatility proportional to short rate), or proportional to the square root of the short rate. Inclusion of mean reversion lessens the impact of this choice because it reduces the variation of the short rate around a long-run value, and therefore reduces the variation of the volatility.

A third issue for interest rate modeling is a bit technical, but comes down to whether the model explicitly determines only the motion of the short rate or describes the motion of the entire yield curve. Models...
expressed solely in terms of the short rate require both a fitting step to match the model to the initial spot curve, and also some approximation scheme or other trickery to find par bond yields (for input to a prepayment model, for example) along a simulation path.

The Heath-Jarrow-Morton [1992] (HJM) approach (used in PACE), by contrast, explicitly models the full spot curve at every point in a simulation. As a result, the spot curve fitting step is not required — the model automatically matches the starting yield curve — and future scenario yields can be found explicitly for prepayment estimation. This makes HJM-type models particularly suitable for MBS valuation.

Monte Carlo Analysis

Monte Carlo valuation of fixed-income securities amounts to a more or less direct implementation of Equation (5). Typically, a random number generator within the interest rate model is used to produce a set of equally probable (risk-neutral) rate paths. The corresponding cash flows are generated and discounted at the cumulative one-period rate. The resulting pathwise present values are averaged to obtain the final result.

While easy to program, random number-based algorithms suffer from statistical noise due to the randomness in the generation of the rate paths. The OAS calculations used for this article are made using an alternative technique developed at CMS and used in PACE for generating paths without random numbers, called the Representative Path (RP) method. Based on comparisons with a random number-based Monte Carlo analysis, the RP method provides a speed enhancement of around a factor of 25 to achieve a given accuracy in CMO calculations. The OAS calculations were done on a low-end Sun workstation using sixty-four interest rate paths, taking anywhere from five to forty-five seconds, depending on the deal and tranche, and giving an OAS valuation uncertainty of within \( \pm 5 \) bp.

OAS and OAY Conventions

There are two conventional definitions of OAS: one generally used in the corporate market, the other in the MBS market. In most cases the actual values given by the two computation methods are the same to within a few basis points, but the use of the two methods and the
relation between OAS and option-adjusted yield (OAY) is a persistent, if minor source of confusion. Both methods use a present value algorithm such as a Monte Carlo or a differential equation solver using as input a yield curve \( Y(t) \) and giving as output the constant spread \( \Delta S \) that has to be added to the yield curve to make the model price equal to the market price.

For corporate and Treasury bonds, the input yield curve is the nominal yield curve for option-free debt of the issuer. Callable corporate debt is thus valued by comparison to non-callable debt of similar maturity from the same issuer. Sometimes a sector yield curve is used instead, but the principle is the same.

The yield curve is shifted by the amount necessary to price the bond. This determines a discount curve in terms of the implied yield curve \( Y(T) + \Delta S \). Using this discount curve, one computes the present value of the “underlying” bond with fixed projected cash flows (e.g., the cash flows to maturity or to call), to obtain the implied price of the bond without the embedded option. Finally, one calculates the yield of the underlying bond at this price (the OAY), and obtains the OAS by subtracting the corresponding maturity Treasury yield.

This definition of OAS corresponds directly to the usual notion of spread to Treasury. For a callable bond at a deep discount, the OAS and nominal spread to maturity will be the same, while for a callable bond at a substantial premium to its first call price the OAS and nominal spread to call will be the same.

A minor problem with this method is that the specification of the underlying bond is somewhat arbitrary. A thirty-year bond with a single European call in ten years may be thought of as either a callable thirty year bond or as an extendable ten-year bond. In general, the OAS computed to maturity will not be the same as the OAS computed to call. Either OAS, however, will still be comparable to the corresponding nominal spread.

The conventional definition of OAS for mortgage-backed securities takes the Treasury curve as the input yield curve. The spread shift \( \Delta S \) is then interpreted directly as the OAS; no yield calculation is involved. The analogue here of the bond OAS method would involve projecting some fixed cash flows for the MBS (say, PSA model cash flows at a reasonable projected speed); discounting these cash flows using the shifted yield curve \( Y(T) + \Delta S \); computing the cash flow yield based on this present value; and finally subtracting a corresponding Treasury yield to obtain an OAS.
The disadvantage of the MBS convention for OAS as compared to the corporate bond convention is that it does not provide a means to vary the option-free discount spread as a function of cash flow timing. As a general rule, option-free spreads on longer securities are wider than those on shorter ones. Corporate bond option models include term structure of spreads among the inputs. On the other hand, this method does not require the arbitrary choice of an underlying option-free security.

For corporate bonds there is usually an obvious choice, e.g., the cash flows to maturity. For MBS other than perhaps unbroken PACs there is no natural basis for specifying the underlying bond. There is, however, no barrier to analyzing MBS with the corporate bond OAS convention, using, for example, constant PSA cash flows to define the underlying security. As with the choice of underlying bond for a callable corporate bond, this amounts to specifying a fixed option exercise.

Analysis of MBS using the corporate OAS convention permits use of a non-constant option-free spread curve. This may be particularly useful for analysis of relatively stable tranche types, for which reasonable estimates of option-free spreads as a function of maturity can be found.

Another application is the analysis of floating rate tranches. For example, with the LIBOR curve as the input, one can determine option-adjusted discount margin for a floater as the shift to the LIBOR spread curve required to price the bond.

If the same fixed cash flows are used to calculate both nominal and option-adjusted yield, then their difference provides a measure of the time value of the embedded option. This is the part of the option cost due to the uncertainty of future interest rates. In the absence of volatility, the option’s time value is zero, but the option’s intrinsic value (based on some predicted exercise of the prepayment option) is unaffected.

Interestingly, an inappropriate choice of cash flows can lead to an OAY greater than the nominal yield — an apparent negative opinion time value. For high-coupon MBS, PSA model cash flows are often inappropriate in this sense. The reason is that, while the prepayment model used for Monte Carlo calculation typically incorporates burnout, resulting in prepayment speed declining over time, the PSA model projection is constant over time. In this case, the fixed PSA cash flows may correspond to a more efficient exercise of the prepayment
option than is predicted by the prepayment model along the typical interest rate scenarios, resulting in an apparent positive option value to the investor. This is illusory: Given a more reasonable prepayment projection to define the “option-free” underlying bond, one would properly observe a negative option time value.

It turns out that, given a suitable definition of the corresponding Treasury yield, the two OAS conventions result in almost the same numeric value. The required Treasury yield is obtained by discounting the same fixed cash flows as used in the OAY calculation (e.g., the PSA cash flows) using the Treasury curve $Y(T)$ instead of the shifted curve.

As shown in the appendix, the OAS defined as the difference between the OAY and this Treasury yield is equal to the spread $\Delta S$ multiplied by a factor very close to 1. The factor is exactly 1 if the yield curve is flat or if the cash flows come in over a narrow time period, and deviates from 1 by a small amount when the yield curve is strongly sloped and the cash flows are widely dispersed in time.

To a good approximation, the two OAS definitions match independently of yield curve slope and cash flow distribution. To use the corporate market OAS convention for MBS, then, one need only make a “reasonable” cash flow projection.

**OAS VERSUS CASH FLOW YIELD ANALYSIS**

I have mentioned that the projection of cash flows based on fixed PSA assumptions can lead to absurd valuations, because the PSA convention is often an unreasonable forecast of the time dependence of future prepayments. The quoted spread used for fixed PSA pricing bears no relationship to any reasonable expectation of actual relative return likely to be realized over the life of the tranche. The nominal spread is merely an “index,” which may sometimes be useful as a rough measure of value compared with other similar tranches and recent trading history, but is not a quantitative measure of value for any but the most stable tranches.

Some investors now use vector analysis to try to overcome this difficulty. Vector analysis is simply the use of non-constant prepayment scenarios to project cash flows. A typical application of vector analysis is to develop prepayment projections under various interest rate scenarios (e.g., rates rising and falling by 100 bp, or a “whipsaw” with both a fall and a rise); using these projections and a
market price, scenario yields are obtained and used to judge the attractiveness or riskiness of the security. The use of prepayment vectors overcomes one of the problems with fixed PSA projections, namely, the improbability of the fixed PSA prepayment scenarios.

Valuation

While prepayment vectors provide more flexibility than the fixed PSA rate convention for determining tranche value, they are still of limited value for analytical purposes. Use of vector cash flows to calculate yield is directly analogous to yield-to-worst analysis of callable bonds: It provides intrinsic value analysis of the embedded prepayment option and, if combined with interest rate scenarios, of other option-like features such as inverse floaters. (The intrinsic value of an option is its present value if future volatility is assumed to be zero.)

Quantitatively, its main value is in the analysis of tranches with relatively short lives whose value is highly dependent on short-run prepayment behavior. For such tranches a short-run projection based on recent prepayment experience may be of more value than an option-based calculation using longer-term scenario prepayment forecasts. Vector analysis can also provide information about, for example, break-even conditions — those presumably extreme scenarios under which the realized yield becomes zero.

Like nominal spread analysis based on fixed PSA assumptions, however, vector analysis is of little use as a quantitative probe of value for most securities. Because it does not provide any measure of option time value (volatility exposure), vector analysis ignores a potentially substantial pricing component. Even tranche types designed to minimize prepayment risk such as PACs and floaters generally show a significant option time value.

For example, the recent issue FNMA 93-120 K in Exhibit 1, a fifteen-year PAC with an effective collar of 90 to 250 PSA, has an option time value of approximately 7/8 point, for an annualized cost of about 10 bp per year under reasonable volatility assumptions. (The calculations in this section are based on prices and Treasury yields from November 1993.) Vector analysis could easily be used to find the range of reasonable scenarios within which the collar is maintained, and those for which it breaks, but it would be difficult to get a quantitative option cost estimate using this information.
Given the collateral WAC of 7.60, this bond will break its band under plausible interest rate scenarios. The cost of this risk can be seen graphically in Exhibit 1, which shows nominal (fixed PSA rate) yield as a function of interest rate change relative to the yield curve under parallel shifts.

The three curves show calculations based on constant OAS (using the RP method in PACE), constant nominal spread with cash flows based on lifetime PSA projections, and constant nominal spread with vector cash flows derived from the CMS prepayment model at each rate level. The vector prepayments are derived assuming the yield curve remains fixed, after imposing the parallel yield shift.4

EXHIBIT 1
NOMINAL SPREAD VERSUS PARALLEL YIELD CURVE SHIFT
FNMA 93-120 K PAC I