n the first of this series of two articles, the case was made for market neutral investing. The central arguments included double alpha, flexibility to diversify risk, an increased opportunity set and most importantly, superior information ratios. Theoretical evidence, Monte Carlo simulations and empirical results all highlighted opportunities to improve performance through an institutional market neutral strategy.

As the previous article suggested, long-short managers and active, long-only managers share many similarities; their methods of portfolio construction reinforce this point. This article discusses the construction of market neutral portfolios using the new long-short optimization feature in the Aegis Portfolio Manager™. We also compare different long-short construction methods to demonstrate the added value of this new feature.

Long-Short Optimization in Aegis Portfolio Manager

Assume Sally is a manager at Laurel & Hardy L.L.P. She has been recruited to construct a $100 million long-short portfolio from cash. Laurel & Hardy’s institutional clientele request a fixed 2:1 leverage, dollar neutrality, i.e., the long positions equal the short positions in value, and beta neutrality. The S&P 500 index is Sally’s proxy of the market as well as her investible universe.

Sally researches several valuation strategies. She finds the Predicted Earnings-to-Price model in the Aegis Alphabuilder® adds value and decides to generate her alphas based on this valuation model. She first truncates her raw signals at three standard deviations. She then converts the raw signals to alphas, based on a medium IC (.05) and the predicted sigma, as illustrated in figure 1.

figure 2 shows Sally’s initial cash position in the Aegis Portfolio Manager workspace, it also shows the S&P 500 as her market and the other tabs available to define her inputs and preferences. figure 3 displays the Optimize settings tab, with long-short selected as the optimization type, the S&P 500 as the universe and leverage fixed at 100% on either side, or 2:1 leverage with dollar
Part Two: The Mechanics of Market Neutral in the BARRA Aegis System™
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EMU Market Override
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These include Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.


The likelihood of default or exit from the EMU differs significantly among these countries now that they no longer print their own money.

Historical data is an essential ingredient to risk forecasts. However, a structural change in markets can abruptly bring about a new regime. In this situation, standard models may need modification and the role of historical data in risk forecasts may change.

In January 1999, a structural change in the European economy took place. The euro, a new currency issued by the European Central Bank, became the principal currency of 11 European countries. It replaced 11 unpegged "legacy" currencies, thus eliminating currency risk within the European Monetary Union. This leaves open the question of whether the EMU should be treated as a single, homogenous bond market or as a collection of legacy markets. Cosmos Global Risk Manager 2.4 facilitates the choice of either paradigm. This article examines some of the implications of that choice.

In November 1998, BARRA released Cosmos Global Risk Manager, version 2.3. This version introduced BARRA's sovereign EMU term structure and local market risk factors. The term structure is estimated from the EMU sub-index of the JP Morgan or Salomon Smith Barney World Government Bond Index. EMU risk factors are based on changes in the EMU term structure and spreads.

The EMU model was intended to augment rather than replace the legacy sovereign models. It was designed as a benchmark against which other EMU term structures are measured. In addition, it serves as a tool to analyze EMU non-sovereign bonds. As of Autumn 1998, our data showed that differences among EMU sovereign markets were significant. There were large spreads between EMU-member term structures and these spreads widened considerably in response to the global currency crisis. Market moves were not highly correlated. Term structure volatility differed across markets. For example, Italy had a much higher term structure volatility than Germany.

The differences are not surprising. The spreads reflect the different levels of perceived credit-worthiness among the EMU sovereigns. High volatility forecasts in Italy...
were generated, at least in part, by the large market moves that took place while Italy strove to meet the Maastricht Treaty criteria.

Furthermore, while monetary policy is controlled by the European Central Bank, control over fiscal policy still rests with sovereign governments. Taxation of capital flows varies by country. Market structure clearly pointed to at least partial independence of sovereign EMU bonds markets, and this was corroborated by the data.

These considerations led to the design of Global Risk Manager version 2.3 valuation and risk models. EMU sovereign bonds are mapped to the legacy market term structures and risk factors. European corporate bonds are mapped to the EMU term structure and risk factors (together with a compatible credit model). The latter decision reflected the fact that the euro was meant to expand and homogenize the European business community. Similar companies in the EMU-zone would have a similar cost of capital, independent of country of domicile. In addition, many have significant operations in multiple geographical regions dispersed across the EMU-zone.

More than a year has passed since the introduction of the euro and there are still strong country dependent effects present in the data. (See figures 1-4.) The persistence of these effects may cause some investors to believe that there will never be a homogenous EMU government bond market. Other investors may believe that a more homogenous EMU sovereign bond market is imminent. This second group will want an investment strategy that treats EMU members as components of one large market with one set of factors. With this in mind, Global Risk Manager version 2.4 has been enhanced with an EMU local market override. This feature enables the user to choose whether to map EMU sovereigns to legacy models or to the EMU model.

The implications of this choice are illustrated in figure 5. Imagine tracking the
Figure 1b
Term Structure of Interest Rates for Major EMU Markets: September 30, 1999

Figure 2
5-Year Spot Rate Spreads Over EMU
EMU Government Bond Index with the sub-portfolio of Italy-issued euro-denominated bonds. The Italy portfolio will have a higher risk forecast in the legacy model than in the override model since the Italy term structure is more volatile than the EMU term structure. On the other hand, the EMU portfolio has roughly the same risk in the two models. This may seem surprising at first, since legacy market shift volatilities tend to be higher than EMU shift volatility. The explanation lies in figure 4, which shows that while EMU and Germany shifts are almost perfectly correlated, the correlation of the Italy shift with EMU or Germany is .65. Similarly, other legacy markets are not perfectly correlated with Germany and EMU. Hence, the diversifying effect offsets the increased volatility of legacy markets.

The most striking difference between the two models appears in the tracking error, which goes from 243 basis points in the legacy model to 21 basis points in the EMU override model (See figure 5A). This is because the European Monetary Union is a single market in the EMU override; therefore Italy bonds are good surrogates for “similar” euro denominated bonds issued by any EMU sovereign. By contrast, Italy issues are poor substitutes for Germany or France issues in the legacy model.

An attempt to track the EMU Government Bond Index with the sub-portfolio of Germany issued euro-denominated bonds yields similar results (See figure 5A). Since shift volatilities for Germany and

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4 The October 31 shift volatility estimates are 102 basis points for Italy, 65 basis points for Germany and Euro.
EMU are virtually identical, the Germany portfolio risk is nearly the same in the two models. Once again, the tracking error decreases sharply from 136 basis points to 17 basis points moving from the legacy model to the override model.

Conclusion: Treating EMU sovereigns as part of one consolidated government bond market, as opposed to retaining legacy factors for modeling purposes, has a material affect on risk estimates of portfolios heavily weighted in those bonds. While market data currently indicates that there are significant differences between legacy markets, many investors preempt further convergence by considering them as one.

Only the passage of time will conclusively determine which mapping paradigm better explains market behavior. Once a sufficient history has been created, out-of-sample tests can be used to compare the models. In the meantime, the users of Cosmos Global Risk Manager 2.4 have the flexibility to impose their view of the structure of the EMU sovereign market on the specification of model exposures.
The Mechanics of Market Neutral in the BARRA Aegis System™
continued from cover page

neutrality. To find a risk aversion compatible with her leverage, Sally performs a standard frontier optimization with no benchmark and no constraints. She opens the optimal portfolios along the frontier until she encounters the portfolio with 2:1 leverage which has a risk aversion of 0.225.

When the long-short optimization type is selected, Sally can constrain independently the beta, the sectors, the industries, and the risk index exposures on the long, the short and the net portfolio. Figure 4 shows a constraint on the beta of the long portfolio between 0.95 to 1.05. To ensure beta neutrality, the net portfolio is also constrained between +/-0.005. Similarly, a maximum number of assets can be specified, and Sally limits the long and the short portfolios to 75 assets each.

At last, Sally has input her alphas and specified her market, universe, leverage, dollar neutrality, beta neutrality and her explicit asset paring constraints. She now optimizes within the Aegis Portfolio Manager. Figure 5 displays the portfolio summary of the optimal market neutral portfolio with an expected information ratio of 1.45 and a net portfolio beta close to zero.

The Aegis Portfolio Manager integrates the Optimizer and the Risk Manager into one comprehensive portfolio management tool. For example, Sally can adjust a parameter such as risk aversion and rerun the optimization without closing, naming or saving her current optimal portfolio. Iterative changes can be explored and different optimized portfolios compared easily within the workspace.

Comparison of Long/Short Construction Methods

While the long-short optimization is a new feature in the Aegis Portfolio Manager (APM Method), the underlying algorithm is the same as that used in HedgeOps.
Seminars & Workshops

United States

Aegis Portfolio Manager and Optimizer Workshop

June 13 | New York, NY
June 27 | Chicago, IL
July 11 | Boston, MA
July 25 | San Francisco, CA
August 1 | New York, NY
August 22 | Boston, MA
August 29 | Austin, TX

This one-day interactive workshop introduces the BARRA Aegis Portfolio Manager and Optimizer and takes attendees through practical hands-on exercises to gain insight into using Multiple Factor Models and related risk analytics for more informed investment decisions.

Contact: Marsha Starr-Fairconeture
Tel: 212.804.1518
Email: marsha.starr@barra.com

BARRA's 24th Annual Research Seminar

July 16-19 | Pebble Beach, CA

BARRA's Annual Research Seminar presents results from BARRA research over the past year into U.S. and international markets, including issues of particular importance for investment managers and researchers. Topics include:

How to Research Active Strategies (Optional Training Course)

Based on the book Active Portfolio Management, this course covers the active management framework, valuation models and expected returns, forecasting and information analysis, portfolio construction and transactions costs, in a combination of lectures and student exercises. This course will run from 8:00 a.m. to 5:30 p.m. on Sunday, July 16 and requires a separate registration and fee.

The New Approach to Global Equity Risk

BARRA is building G2, the new global equity risk model. Our current GEM model analyzes risk primarily in terms of country bets. G2 focuses more on the granular structure of industries and risk indices within countries. Why have we chosen this new approach?

Estimation of the G2 Model

This talk will examine modeling issues related to estimating the G2 global equity risk model. Preliminary historical estimation results will be presented for comment.

Currency Risk Modeling

Currency risk accounts for a big fraction of the risk in an international portfolio. Find out how BARRA's statistical currency risk forecasting model performs against an implied volatility model.

Insights from G2

In this section, we present several case studies that reveal how the new global equity model has enhanced our understanding of global portfolios.

U.S. Fixed Income Modeling

Risk models for fixed income securities have assumed a higher profile in the aftermath of the 1998 credit crash and the ensuing market liquidity premium. This presentation introduces BARRA's new U.S. fixed income risk model, which includes a factor model of interest rates and spreads, as well as a credit migration model for issuer risk.

Credit Risk in Europe

How has the euro affected fixed income investing in Europe? We'll look at sovereign and corporate credit spreads as well as the growing Pfandbrief market.

Multi-Factor Valuation

Valuation of embedded options makes fixed income portfolio risk forecasting very time consuming even on the fastest hardware. Although multiple factors drive interest rate movements, one-factor approximations are often used to speed processing. Compared to a two-factor model, what are the practical consequences of choosing speed over accuracy?
Testing Cross Asset Class Risk Forecasts
Forecasting risk for the firm as a whole requires consistent aggregation across equity, fixed income, and other asset classes. We will examine the tools BARRA has developed to make such forecasts and evaluate their performance.

Optimal Rebalancing of Correlated Asset Classes
This talk develops an optimal rebalancing strategy that avoids both the excessive costs of trading too much and the unacceptable tracking error of trading too little. The theory gives satisfying results when realistic levels of correlation are introduced among the asset classes.

Market Impact and Momentum
Market impact costs accrue to an order when the price drifts before the order is filled. They afflict managers whose strategies tend to follow the market, and can equal or exceed losses due to price impact. Using trading data we characterize managers' susceptibility to momentum costs, and show that total transaction cost can be reliably forecast.

Strategic Asset Allocation for Pension Plans
New title: "Strategic Asset Allocation for Pension Plans" What objectives and risks do pension plans face when making strategic asset allocation decisions in the context of asset liability management? This presentation introduces a methodology for asset allocation under risk budget constraints.

Risk Horizons and Specific Risk
Recently, global equity markets have witnessed increased levels of risk, as well as increased variability of risk. How have BARRA models captured this increase in risk? Is it transient or permanent? How should portfolio managers respond to these events?

After Tax Investing
Taxes present a challenging hurdle to active management. After analyzing the factors that detract from after tax performance, we present an optimization-based tax conscious approach to actively managing portfolios.

Risk Control and Transaction Costs
This talk will present case studies addressing the tradeoffs between alpha, tracking error, and transaction costs. What is the typical level of transaction costs incurred in order to maintain tight risk control? By how much will transaction costs be reduced if tracking error bands are employed?
Modeling Analyst Skill

The timeliness of an analyst’s earnings forecast can be as important as its accuracy. We propose a measure of analyst skill that accounts for both accuracy and timeliness, and explore its dependence on prior skill, investment banking relationships, experience, and other factors.

Behavioral Finance

This talk explores the trading behavior of individual investors. How do they choose which stocks to buy and which to sell? Why do they trade too much? How does the trading of men and women differ? Why do investors trade more actively once they go online?

Discovering Hidden Risk States

How can we improve our forecast of the full probability distribution of returns? We present switching regimes models (also called hidden Markov models) and compare their predictive abilities to currently used methods.

Contact: Rosalie Javier
Tel: 510.649.4265
Email: rosalie.javier@barra.com

Equity Portfolio Management Seminar

September 24-27 | Chatham, MA

This seminar focuses on an introduction to the investment and statistical concepts that underlie BARRA’s equity portfolio management products. New to the program this year is the addition of an optional extra day of applied theory and one-on-one consultation.

Contact: Sherri Roberson
Tel: 510.649.6481
Email: sherri.roberson@barra.com

International

BARRA’s Introduction to Equity Risk Workshop

June 7 | London, UK
July 5 | London, UK
August 9 | London, UK
September 6 | London, UK

This half-day workshop aims to give a basic introduction to BARRA and the theory behind BARRA’s Equity Risk Models. This program will also review the BARRA Aegis Risk Manager for a UK portfolio.

BARRA’s Fixed Income Workshop

July 6 | London, UK

This workshop assumes basic knowledge of Fixed Income risk measurement techniques. We focus our presentation on the theory that drives BARRA’s Fixed Income modeling and compare this to other methodologies. A session focused at analyzing a Global Fixed Income portfolio will follow, which will give a general overview of the analytics available through Cosmos Global Risk Manager.

BARRA’s Return Forecasting and Optimization Workshop

June 8 | London, UK
September 7 | London, UK

This half-day workshop covers the theory behind valuation models, raw forecasts and alphas - refining returns, sensitivity analysis, optimization techniques and portfolio construction. An overview of the BARRA Aegis System Optimizer will include an introduction to the optimizer’s features (i.e., constraints, penalties), how to generate a minimum tracking error portfolio, and how to generate an efficient frontier of portfolios.

BARRA’s Performance Attribution Workshop

August 10 | London, UK

This half-day workshop covers BARRA risk theory recap, structure for returns attribution, single month analysis, cumulative analysis, performance evaluation and information ratios. A case study using the BARRA Aegis Performance Analyst will include creating a performance database, analyzing a portfolio’s performance, risk and return attribution and evaluating a manager’s skill.

The contacts for all seminars offered in London:

Penny Beaufrere/Melissa Shuckard
Tel: 44.171.283.2255
Fax: 44.171.220.7555
Email: penny.beaufrere@barra.com
melissa.shuckard@barra.com

Research Seminars

An international version of the Annual Research Seminar is being planned for London, Sydney, Tokyo and Singapore in October/November. Check our website in the coming months for further details.
The APM Method executes three sequential optimizations to determine the optimal portfolio. First, an optimization with no constraints is performed to estimate an implied risk aversion given the leverage. Second, an optimization is performed with the implied risk aversion and the additional constraints with the exception of asset paring, leverage and the constraints unique to the long and short portfolios. This second step parses the investible universe into long and short candidates. The optimizer then performs a final optimization with the user’s risk aversion and constraints.

Before the long-short optimization was introduced in the BARRA Aegis System, two other methods were widely used by long-short managers: the Iterative Method and the Alpha Parsing Method.
The Iterative Method requires the user to run multiple optimizations. First, a long portfolio is built with all the alphas, all the members of the investible universe and a benchmark. Second, to create the short portfolio, the sign of the alphas is reversed, assets in the long portfolio are removed from the investible universe and the long portfolio is specified as the benchmark. Finally, the short and long portfolios are combined to form the final long-short portfolio.

The Alpha Parsing Method executes one standard optimization to find the optimal portfolio. The investible universe is parsed by the sign of the alpha. A positive alpha denotes a long candidate while a negative
alpha denotes a short candidate. No benchmark is specified, and the long and short portfolios are created simultaneously.

Graph 1, “A Comparison of Long-Short Construction Methods,” illustrates the efficient frontiers produced by these three methods. These frontiers were constructed with Sally’s alphas, the S&P 500 as the market and the universe, 2:1 leverage, dollar neutrality and no additional constraints.

The APM method produced the superior frontier or the best risk and return combinations. At low levels of risk, the APM method adds significantly to the expected information ratio. At high levels of risk, the Alpha Parsing frontier converges to the APM frontier since an aggressive strategy emphasizes optimal return combinations. The Alpha Parsing method is less efficient at low levels of risk because it parses the investible universe without consideration of the risk dimension.

The Iterative frontier is the most inferior of the three methods, however, further iterations will shift the Iterative frontier closer to the APM frontier.

The superiority of the APM method is accentuated by transaction costs, an initial portfolio other than cash, and additional constraints (such as beta neutrality or asset paring).

If managers are not dissuaded by the inferior Alpha Parsing and Iterative solutions, they should also consider the difficult implementation issues related to these methods. The Iterative Method does not permit a manager to explicitly constrain the net portfolio since the long and short portfolios are created separately. The Alpha Parsing Method does not allow a manager to constrain the long and short portfolios independently or constrain the beta of the net portfolio.

Users of HedgeOps will find that the Aegis Portfolio Manager holds additional important advantages. The Market Impact Model™ (MIM), available in the BARRA Aegis System, forecasts non-linear, ex-ante transaction costs. Also, Value at Risk (VAR) and Return at Risk (RAR) capabilities in the BARRA Aegis System permit market neutral or hedge fund managers to estimate their capital requirements.
The BARRA Brainteaser: hotair.com

by Kenneth Hui

At a dinner party for money managers, the discussion turns to whether hotair.com, a recent IPO that uses the internet to arrange hot air balloon rides, is over-valued. Every manager is asked to give the company a thumbs up or a thumbs down. None of the 10 money managers seated at the round table really knows much about the company, but no one wants to appear uninformed, so everyone has the tendency to agree with their two immediate neighbors, without regard for their own opinion. In fact, the relative probability of observing a particular realization or configuration of opinions around the table depends only on the number of disagreements between the 10 pairs of neighbors.

A configuration with no disagreement is 4 times more likely than a configuration with 2 disagreements, and 16 times more likely than a configuration with 4 disagreements. In general, a configuration with no disagreement is $2^N$ times more likely than another configuration with $N$ disagreements. Note that the situation in which all managers except manager A give a thumbs up is counted as distinct from the configuration in which all managers except manager B give a thumbs up.

On average, how many pairs of neighbors agree in their opinion of hotair.com?

The following mathematical formula for the trace, or the sum of the diagonals, of the product of $N$ matrices $M$ might be useful:

$$\text{Tr}(M^N) = (x + x^{-1})^N + (x - x^{-1})^N$$

Brainteaser from last issue

by Mark Ferrari

Problem

The trading community is buzzing with the rumor that the upcoming Star Wars movie will offer a glimpse of the capital markets that finance the Galactic Empire's expansion. One scene reportedly takes place on the floor of the Imperial Stock Exchange (ISE), an oceanless planet whose entire surface has been covered with sturdy blue carpeting and fluorescent lighting. Scattered randomly on the surface of this planet are outposts of the galaxy's various securities firms which function as dealers; there are no specialists on the ISE. Due to the complete lack of natural topography, any location on the planet is equally likely to be the site of a dealer's post. Dealers trade with each other by dispatching order-carrying robots. Trading electronically would be much more efficient, but that would leave the Hollywood special effects wizards with little to depict.

Consider the post of Dealer A, who is sending out his robots in random directions in search of counterparties. Because the other dealers' posts are small (for the purposes of this problem you may assume...
they are geometric points), the robots risk circumnavigating the planet many times before blundering into one. Accordingly, they are programmed with the following rule - the post closest to the robot must at all times be either Dealer A (its origin) or the post at which it will arrive if it continues without turning, which we will call Dealer B. If the robot detects a third post (Dealer C) which is closer than either of these - call this event a close encounter - it returns home to Dealer A, registers Dealer B as an unacceptable destination to prevent other robots from wasting their time on it, and sets off for Dealer C. The effect of this rule is that one dealer will trade with another if and only if a robot traveling directly between them would never find itself closer to a third dealer than to the closer of the two.

Each dealer trades with several other dealers, the exact number of which depends on how the posts happen to be arranged in his neighborhood. If the posts are randomly and independently situated, what is the average number of counterparties with whom each dealer trades? You may assume that the average distance between dealers is much smaller than the size of the planet, so that the curvature of the planet may be neglected. In other words, a flat map is an adequate representation of any part of the globe.

**Bonus Questions**

1. Imagine that Dealer A finds he has too few counterparties to effectively work his trades. He reprograms his robots so that they ignore the first close encounter but turn back upon the second. What happens to his expected number of counterparties? What if he allows his robots to ignore two close encounters?

2. If trading took place in a three-dimensional space rather than on a two-dimensional planetary surface, what would happen to the expected number of counterparties?

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**Brainteaser Solution**

by Mark Ferrari

Let \( N \) be the total number of dealers on the Imperial Stock Exchange, and \( A_r \) the surface area of the planet on which they are located. The mean density of dealers is \( n = \frac{N}{A_r} \). Because the planet is much larger than the average distance between dealers, curvature effects are not important, and we can approximate the ISE as an infinite plane with \( n \) dealers per square kilometer.

Consider Dealer A and Dealer B separated by a distance \( s \). They will trade with each other if, and only if, a robot traveling on a straight line between them is never closer to a third dealer than to the nearer of A and B. This is equivalent to saying that the circle whose diameter (not radius) is the line from A to B contains no other dealers. If Dealer C were located within the circle, then a robot at the center of the circle would be closer to C than to A or B. In Figure 1, C is inside the circle and would cause a close encounter, but D would not.

The area of this circle is \( A = \pi s^2 / 4 \). The expected number of dealers inside the circle is \( \mu = nA = \pi ns^2 / 4 \).

Because of the size of the planet, the probability that any particular dealer falls within the circle is very small. Thus the probability that the circle contains exactly \( m \) dealers is given by the Poisson distribution

\[
p(m) = \frac{\mu^m}{m!} e^{-\mu}
\]

In particular, the probability that the interior of the circle is empty is

\[
p(0) = e^{-\mu} = \exp(-\pi ns^2 / 4)
\]

Now that we know the probability that two dealers will trade, let us investigate the trading opportunities open to Dealer A. Consider an annular region of radius \( s \) and width \( ds \), centered on Dealer A. The area of

![Figure 1](image-url)
this region is $dA = 2\pi s \, ds$ and the expected number of dealers it contains is $dD = n \, dA = 2\pi ns \, ds$. However, because of the empty-circle requirement, only a fraction of these will be valid counterparties; the expected number of dealers in the annulus with whom Dealer A trades is

$$dC = p(0)dD = 2\pi n s \exp(-\pi s^2 / 4)ds$$

The total number of counterparties is the sum over all such annuli:

$$C = \int dC = \int_0^\infty 2\pi n s \exp(-\pi s^2 / 4)ds$$

$$= 8\int_0^\infty ye^{-y^2} \, dy$$

$$= 4$$

where we have used the change of variable $y = \sqrt{\pi s^2 / 4}$. With an entire planet full of dealers, the expected number of counterparties is only four! Note that this is purely a geometrical result, depending on the robots' programming and the two-dimensional nature of the problem, but not on the size of the planet or the number of dealers.

Extensions

It is a straightforward matter to extend this solution to the case where $m$ close encounters are allowed before the robot turns back. Using Eq. 1, the probability that the circle contains $m$ or fewer dealers is

$$P(m) = \sum_{k=0}^{m} \frac{(\pi s^2 / 4)^k}{k!}$$

The number of counterparties in the annulus becomes $dC = 2\pi n s P(m) \, ds$. As before, we sum over annuli, this time using the identity

$$\int_0^\infty y^{2k+1} e^{-y^2} \, dy = \frac{k!}{2}$$

to obtain the expected total number of counterparties $C = 4(m+1)$. The original problem, in which no close encounters were allowed, is the special case of this result with $m = 0$. When one close encounter is allowed, the number of counterparties rises to 8, and with two encounters it is 12 - simply linear in $m$.

If the ISE were to outgrow its planetary home and expand into space, the problem could also be solved for a volume density $p$ of dealers per cubic kilometer. The region which must be empty for two dealers to trade is now a sphere of volume $V = \pi s^3 / 6$ and the expected number of dealers in this volume is $\mu = pV = \pi ps^3 / 6$. The probability that $m$ or fewer dealers fall within it is

$$P(m) = \exp(-\pi ps^3 / 6) \sum_{k=0}^{m} \frac{(\pi ps^3 / 6)^k}{k!}$$

The volume surrounding Dealer A is analyzed as a set of spherical shells of volume $dv = 4\pi s^2 \, ds$ each containing an expected number of counterparties $dC = 4\pi ps^2 P(m) \, ds$. The total number of counterparties in this case is

$$C = \int_0^\infty 4\pi ps^2 \exp(-\pi ps^3 / 6) \sum_{k=0}^{m} \frac{(\pi ps^3 / 6)^k}{k!} \, ds$$

$$= 24 \sum_{k=0}^{m} \frac{1}{k!} \int_0^\infty 2^{2k+2} e^{-z^3} \, dz$$

$$= 24 \sum_{k=0}^{m} \frac{1}{k!} \frac{k!}{3}$$

$$= 8(m+1)$$

using the substitution $z = s (\pi p / 6)^{1/3}$. If the robots turn back at the first close encounter, the expected number of counterparties in the three-dimensional problem is eight. If one or two close encounters are allowed, the numbers of counterparties are 16 and 24 respectively - again linear in $m$. ■
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In compliance with applicable federal regulations, BARRA hereby offers to each of its advisory clients a copy of Part II of its most recent Form ADV. To obtain a copy, please call or write Maria Hekker at BARRA, 2100 Milvia Street, Berkeley, CA 94704, Telephone 510.548.5442. There is no charge for the document.

2100 Milvia Street
Berkeley, California 94704-1113
U.S.A.