The Barra fixed income risk model ascribes bond returns to two types of local market risk factors (aside from exchange rates). They are return due to term structure changes and return due to spread changes.

Term structure changes are represented in terms of shift, twist and butterfly (STB) factor shapes $\phi_i(T)$, obtained by principal component analysis of the covariance matrix of spot rate changes in each local market. (The shapes are smoothed to take out high frequency components, and adjusted on a monthly basis to make their model correlations vanish.) The factor shapes $\phi_i(T)$ are normalized so that (1) they are positive at maximum $T$, and (2) their norm is equal to the number of vertices, or maturity points.

The exposures to the STB risk factors are derivatives of bond price with respect to term structure change

$$E = -\frac{1}{P}\frac{\partial P(r(T) + \theta \phi_i(T))}{\partial \theta},$$

where $P$ is the present value of the bond, $r(T)$ is the interest rate curve or term structure, $\theta$ is the amount by which the term structure is shifted and $\phi_i(T)$ is the $i^{th}$ STB factor. The numerical calculation is a two-point finite difference:

$$E = -\frac{1}{P}\frac{P_{up} - P_{down}}{2\theta}$$

where $\theta$ is the amount that the interest rates are adjusted (by default, 25 bp), and $P_{up/dn} = P(r(T) \pm \theta \phi_i(T))$. Barra systems define term structures by their values at the vertices. Interpolation between vertices is based on assuming constant forward rates. The scaled STB factor shapes are added to the spot rates at the vertices. Then new forward rates are found to provide the required interpolation and produce the adjusted term structure.

Exposure to spread change is defined more straightforwardly:

$$E = -\frac{1}{P}\frac{\partial P(r(T), s)}{\partial s}$$

where $s$ is the bond spread. Depending on the particular asset, this may be evaluated numerically or analytically. If numerically, the shock size is again 25 bp by default.