The Barra
Multiple-Horizon Equity Model™

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for the document.
In this issue of Horizon, we introduce a major innovation in equity risk modeling: the Multiple-Horizon US Equity Model. Equity markets have become highly dynamic in recent years, complicating the task of forecasting risk. Guy Miller, Barra’s Manager of Equity Research, and Fang Wang, Senior Research Consultant, reveal how the Multiple-Horizon Equity Model produces more responsive and accurate risk forecasts by incorporating daily factor returns and investor time horizon into the proven factor structure of the US Equity Model. A second article, by Senior Consultant Edouard Sénéchal, discusses how changes in US equity market volatility and correlation have impacted predicted beta in the US Short-Term Model.

Continuing our exploration of credit risk, Oren Cheyette, Barra’s Vice President of Fixed Income Research, examines the empirical relationship between a credit-risky bond, the return to the issuer’s equity and changes in default-free interest rates.

A fourth Insights article concludes our series on the Barra Integrated Model. Focusing on the equity portion of the model, we discuss the practical benefits that investment managers derive from the combination of breadth and depth delivered by the Barra Integrated Model.

Horizon’s regular features include a pullout calendar of Barra’s upcoming events, a section highlighting recent publications by Barra’s research group, and the popular challenge of the Barra Brainteaser.

Barra Horizon is also available on the web at http://www.barra.com/horizon.
There has been great interest in recent years in obtaining improved quantitative understanding of credit risk. Recent portfolio shifts from equities to bonds, increased corporate bond issuance, the well-publicized defaults of several large issuers, and a continuing surge of interest in credit derivatives have motivated a substantial investment in research and model development.

The prototype for much of this research is the “structural” Merton (1974) model. Alternative, more empirically motivated models are based on observed credit migration rates (“reduced form” models), or combine empirical data with information from structural models (e.g., RiskMetrics’ CreditMetrics model). Structural models have had some commercial success. There is good evidence that they provide more accurate forecasts of default probabilities than do agency ratings. In principle, structural models should also be capable of providing information about correlations of bond returns (short of default) and correlations of bond and equity returns, derived from predictions of firm value correlations. These correlations are clearly of central importance to understanding risk in a portfolio context.

In practice, applications of structural models to problems of valuation and portfolio risk have not had great success. Partly, this is because structural models do a poor job of fitting bond price and return data “out of the box”. A typical model needs to be calibrated to empirical default rates and bond spreads in order to be useful as a market model. Once this has been done, such a model is no longer genuinely predictive, since its forecasts have been modified to fit historical or current observations. Instead, it can be thought of as providing a plausible basis for interpolating and extrapolating the calibration data, based on factors relevant (according to the model) to default rates and/or bond spreads.

There are also a number of practical difficulties with their application, including a need for accounting data that may be out of date and difficulty handling realistic capital structures—particularly for highly leveraged financial firms. Moreover, an investor with mark-to-market portfolios holding public securities will probably be at least as interested in market returns as in the specific event of legal default.

Defaults don’t usually happen out of the blue:1 by the time legal or de facto default has occurred, affected securities will have been

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1 When they do, it is usually because of fraud, which is not realistically detectable by any quantitative method.
marked down to prices reflecting the expected recovery. For purposes of risk modeling—if the goal is to forecast the volatility of asset or portfolio returns—a calibrated structural model may be sufficient, but it is not necessary. If we can estimate the relationship between a bond and its issuer’s equity by some other means, as well as forecast equity return volatility and correlations, then we do not need forecasts of either default probabilities or recovery rates, or their impact on bond value. We can thereby sidestep the difficulties with all the inputs and outputs from structural models, and focus on just those results required for predicting the volatility or distribution of bond returns.

Motivated by these considerations, we describe here a statistical model of the relationship between the return to a credit-risky bond, the return to the issuer’s equity and changes in default-free interest rates. We take market prices (in the form of option-adjusted spreads) as given, using them as the basis for forecasting bond risk in terms of interest rate, equity and residual spread factors. We refer to this approach as “empirical credit risk” (ECR).

Our research reveals some interesting, perhaps not entirely surprising facts about the links between bond and equity markets. First, we have found that a bond’s spread relative to the Treasury curve provides an effective measure of the degree to which the bond’s return will be correlated with Treasury bond returns (i.e., changes in default-free interest rates) and with the return of the issuer’s equity. While previous academic studies have found a weak relationship based on agency rating, we find a strong dependence on market spread. From the standpoint of modeling risk, a significant
advantage of using bond spread is that it is a much more timely and responsive measure of perceived credit quality than agency rating.

Second, after examining a number of factors that might have additional influence on the return relationships, we find two clearly significant effects. Bond duration is found to increase the exposure of bonds to equities at intermediate levels of credit quality, though not for the most distressed issues. Most intriguingly, we find clear evidence that the bond-equity return linkage is significantly weaker for positive equity specific return than for other sources of equity return (common factors or negative issuer-specific events). This might, plausibly, be due to agency effects where management acts to increase shareholder value at the expense of bondholders. On the other hand, we find no clear impact of equity volatility or sample period, and weak evidence of sector dependence. We also see no difference in the behavior of “natural” high yield bonds compared to fallen angels.

We propose to use the empirical relationship between bond returns, interest rate changes and equity returns as the basis for a significantly improved approach to portfolio risk modeling. We show that, in combination with a factor model for the attribution of corporate bond returns to equity and interest rates, we are able to account for anywhere from 45% to 90% of the variation in bond returns, depending on credit quality. This is a substantial improvement on a simpler model that ignores the attribution to equity, but also improves on a model that leaves out the residual credit spread factors, particularly for bonds of intermediate credit quality.

This article describes our analysis and some applications of the results.

Model Structure

We seek to model the return to a corporate bond in terms of the returns to government bonds—i.e., changes in default-free interest rates—and the return to the bond issuer’s equity. We represent this relationship through the return decomposition

\[ r^i_{BSS} = \beta_E r^i_{ISS} + \beta_IR r^i_{GOV} + \epsilon^i_B \]

giving the excess return \( r^i_{BSS} \) to a corporate bond (denoted by the subscript \( B_{SS} \)) in terms of the corresponding excess return \( r^i_{GOV} \) to an equivalent government bond, the equity excess return \( r^i_{ISS} \) and a residual. The coefficients \( \beta_E \) and \( \beta_IR \) measure the dependence of the bond return on interest rate changes and equity returns. The “equivalent government bond return” is derived by treating the bond as default-free, computing the implied interest rate factor exposures (“durations”, with conventional minus sign) and then summing the return contributions from interest rate factor changes obtained (by inversion of this relation) from actual government bond excess returns: \( r^i_{ISS} = \sum (\Delta r^j_I) r^j_{ISS} \). The interest rate factors are given by approximate principal components of the term structure movements.\(^4\)

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\(^2\) Equity specific return is the component of a stock’s return not explained by either passage of time (the risk-free rate) or equity market common factors.

\(^3\) Excess return is total return minus the risk-free rate. The precise specification of excess return is inessential to the results.

\(^4\) Barra’s interest rate risk models incorporate three factors (approximate principal components of the covariance matrix of spot rates) responsible for most of the term structure variation in the 3-month to 30-year maturity range.
For high-grade bonds, the calculation of durations adjusts for the timing of cashflows and other bond structural features, so that when the coefficients of Equation 1 are estimated using the government bonds originally used to find the interest rate changes (and taking $\beta_r = 0$, of course), we obtain $\beta_{\text{IR}} = 1$.

Equation 1 should be contrasted with the return decomposition for the existing fixed income factor model in terms of interest rates and spreads. The analogous equation for the current model is

$$r_t^{\text{iss}} = r_{t}^{\text{gov}} + (-D_t^{\text{spread}}) \Delta s_{\text{CAT/iss}}^t + \varepsilon^t$$

In this model, all bonds have unit exposure to returns of corresponding default-free bonds, and the residuals are captured with sector/rating spreads $\Delta s_{\text{CAT/iss}}$ based on the issuer. In this model, the exposures are all known, and the model tells us what the sector/rating spread returns are, given the bond residual returns after accounting for interest rates. In the empirical credit risk model, by comparison, the explanatory returns $r_t^{\text{gov}}$ and $r_t^{\text{iss}}$ are given, and the return Equation 1 is used to find a model for the exposures $\beta_{\text{IR}}$ and $\beta_{E}$.

The coefficients $\beta_{\text{IR}}$ and $\beta_{E}$ measure the dependence of the bond return on interest rate changes and equity returns. Ultimately, we are seeking functional representations for $\beta_{\text{IR}}$ and $\beta_{E}$ to capture their dependence on credit quality and perhaps other characteristics of the bond or firm in question. Before looking at the data, we can anticipate some qualitative aspects of the relationship between bond prices, returns, interest rate changes and equity returns. We expect the option-adjusted spreads (OAS's) of bonds with high default risk to be large. Accordingly, we anticipate that returns to low-OAS bonds will be primarily determined by changes in default-free interest rates, and nearly independent of equity returns, while returns to high-OAS bonds will be less sensitive to changes in default-free interest rates, and more sensitive to equity returns.

Figure 1

Merton model forms for $\beta_{IR}$ and $\beta_{E}$ as a function of OAS. The equity volatility is 100% annually. (At high OAS, this implies considerably lower firm value volatility.) The bond maturity is 10 years. $B$ and $S$ denote the bond and stock price and $r$ denotes the interest rate. The shapes of these curves are not strongly sensitive to either the volatility or maturity assumption, but the largest realizable value of the OAS does depend strongly on the volatility. The relatively high value of 100% was chosen to span an interesting range of OAS's.

These criteria imply that $\beta_{\text{IR}}$ and $\beta_{E}$ should have roughly the behavior shown in Figure 1. The curves of this figure are derived from the Merton model. They show $\beta_{\text{IR}}$ decreasing from 1 at zero OAS towards 0 at large OAS. Conversely, $\beta_{E}$ increases from 0 at zero OAS to an intermediate value (in this example, slightly greater than 0.5) at large OAS.

Note that because $\beta_{E}$ depends on OAS (or, equivalently, on price), the bond-equity return relationship implied by Equation 1 is non-linear.

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1. The OAS is the amount by which the default-free zero-coupon yield curve must be shifted in order that the bond valuation model reproduces the bond's market price. Our valuation algorithm captures contributions to bond value due to cashflow timing and embedded options, such as calls, puts and sinking funds, but assumes no default. This definition generalized the notion of yield spread of option-free bonds relative to a default-free benchmark bond.
Equation 1 with constant coefficients may be viewed as a locally linear approximation to the full functional relationship. Although we estimate parameters of this restricted (linear) form for exploratory purposes, the full model we seek has more general non-constant $\beta_{E}$ and $\beta_{IR}$.

Data

The regression Equation 1 involves three sets of returns. On the left side of the regression equation are bond excess returns. On the right side are equity excess returns, and "equivalent" government bond returns, or, in practice, interest rate factor returns and calculated exposures of the bonds to the interest rate factors. Our estimation universe, then, consists of a collection of bond and corresponding equity excess returns (typically more than one bond's return per equity return) pooled cross-sectionally and over time and the corresponding interest rate factor changes for each period in the sample.

For convenience, and to minimize problems of synchronization and stale price information, we use monthly returns. Bond data are obtained from the Reuters/EJV fixed income database. This database currently contains approximately 40,000 active US dollar corporate bonds. Many of these are small issues, medium term notes, or other illiquid securities.

In order to improve the quality of the return data, we restrict analysis to those bonds found in a corporate benchmark of a major index provider. We chose the combination of the corporate component of the Merrill Lynch U.S. Domestic Master and the Merrill Lynch U.S. Domestic High Yield Cash Pay indices.\(^6\) By restricting our analysis to this estimation set, we can be confident that we are including only a relatively liquid portion of the bond universe.

Equity data are obtained from Bridge (Reuters) and IDC. We include the estimation universe from the Barra US equity risk model (USE3), consisting of 2000 equity issues, comprising the top 1500 public firms by market capitalization, plus an additional 500 firms chosen to fill thin sectors. The full dataset consists of bond and equity returns over the period beginning January 1996 and ending October 2002 (82 months).

Bond price data are famously problematic. As a test to confirm that bad vendor prices are not causing significant errors in our results, we reran the analysis using prices from an alternate source, namely the Merrill Lynch Index group. Because our estimation universe was already based on the Merrill Lynch Domestic Corporate indices, the only change was in the prices—not in the estimation universe. The results using the alternative prices were qualitatively the same, and quantitatively only slightly different from the results based on Reuters/EJV prices.

Analysis and Results

To build a risk model, we will eventually want to estimate smooth functional forms, qualitatively similar to the Merton model shapes, and giving reasonable asymptotic behavior for both $\beta_{E}$ and $\beta_{IR}$ at low and high OAS. But before estimating this heuristic model, we need to characterize the empirical behavior of the $\beta_{\cdot}$—we need to know what factors are responsible for variation in the dependence of bond returns on equity returns and interest rate changes, and we need to know what the smooth empirical functional forms should look like, at least in the range of OAS's where the results are not too noisy.

Starting with a regression of Equation 1 on the aggregate data binned by OAS range, we then drill down to examine additional dimensions of possible variation. The statistical uncertainty on the regression $\beta$'s is estimated by bootstrap resampling$^7$ with 200 runs. Error bars in the figures and tables are one standard deviation uncertainty estimates from the bootstrap runs.

We take OAS as a basic market measure of credit quality that we expect to affect the regression coefficients. OAS is calculated relative to a default-free zero coupon yield curve fitted to Treasury bill, note and bond prices, although OAS's relative to a swap curve would give fairly similar results.

Aggregate Estimation Results
The results of the regressions on data binned by bond OAS are shown in Figure 2.

![Figure 2](image-url)

The graph shows three series of values: the interest rate $\beta$'s, higher on the left; the equity $\beta$'s, higher on the right, and the regression $R^2$, high on both the left and right, and falling to a minimum around 400 bp OAS. The one standard deviation error bars are too small to see for many of the data points, primarily at low OAS. Measured by the t-statistic, $\beta_{IR}$ is significantly different from 0 at all OAS levels, even the 0-50 bp range. However, equity return contributes substantially to explaining bond return only once the OAS exceeds about 200 bp. The interest rate exposure, $\beta_{IR}$, is not significantly different from 0 for the OAS > 700 bp bins, well into high-yield territory. Correspondingly, in Figure 2 the error bars become too large to fit on the graph, while the best-fit values of $\beta_{IR}$ are off-scale.

One notable feature of this graph is the minimum of $R^2$ in the neighborhood of the investment-grade/high-yield boundary. For bonds with OAS between about 250 and 440 basis points, the model $R^2$ reaches a minimum of about 10%. The implication of this low $R^2$ is that neither interest rate changes, as measured by government bond returns, nor changes in firm value, as measured by equity returns, are explaining much of the bonds’ returns.

A further interesting observation is the rapid drop in $\beta_{IR}$ from near 1 for the lowest OAS bin to below 0.5 for bonds having OAS's above 350 bp, while for bonds with OAS above 700 bp, the estimate of $\beta_{IR}$ is not significantly different from 0. The highest credit-quality bonds have interest rate sensitivity nearly equal to that of Treasury bonds with the same cashflows. On the other hand, interest rates explain essentially none of the returns of low credit-quality bonds. This is consistent with the market “lore,” that duration overstates the sensitivity of lower-grade bonds to interest rates.

Exploratory Analysis
We did further exploratory analysis on additional groupings of the data, by
- bond maturity (duration)
- projected equity volatility

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The first two tests are motivated by considerations from structural models. In particular, we expect that bond maturity and equity volatility should influence the exposures of bonds to equity. (The main effect of maturity on interest rate exposure for high quality bonds is already accounted for by the calculated factor exposures $(-D_{B_i})$.)

The other tests are from sources of variation not predicted by a structural approach. Grouping by sample period tests the stability of the empirical relationship over time. Grouping into positive and negative equity return sets tests, among other things, whether the general phenomenon of “down markets = higher correlations” holds in this area. We also did one further analysis based on a slight extension of the model of Equation 1, splitting the equity return into two components: a term due to the common-factor return, based on the USE3 equity model, and the residual, or specific equity return. A substantial difference in the exposures of bond returns to the two components of equity return would be hard to achieve in a structural model. Except for a few market factors, such as interest rates, structural models are directly sensitive to capital structure and firm value, but do not distinguish the effect of different sources (common factor or firm-specific) of change in firm value.

Grouping by sector is of particular interest for financial firms — as noted in the introduction, such companies’ debt is difficult to model in a structural framework. Finally, we were simply curious whether fallen angels behaved differently than bonds issued with speculative ratings. A positive result might indicate that fallen angels are followed differently by analysts—or by different analysts—than firms whose debt has never been investment grade.

Briefly summarizing the results:

- None of the examined factors strongly affect the estimates of the interest rate exposure coefficient $\beta_{IR}$. A bond’s interest rate exposure depends on bond characteristics as measured by its calculated exposures $(-D_{B_i})$ and its OAS, and not on any other factor examined.

- For OAS in the intermediate range of 100–1000 bp, $\beta_{e}$ is moderately increasing in bond maturity or duration. One way of interpreting this result is that, to some degree, firm-specific news is absorbed into bond prices as a change in bond spread across maturities. Were this uniformly true, with constant spread change independent of maturity, we would find $\beta_{e}$ to be proportional to duration. We actually find weaker dependence than this, implying that—all else equal—spreads of shorter term bonds change more than those of longer term bonds for a given equity return.

- $\beta_{e}$ is not significantly sensitive to equity volatility, sample period, sector, or a firm’s status as a “fallen angel”.

- $\beta_{e}$ depends strongly on the sign of the equity return, with negative return events having a much larger effect on bond return than positive return events. By splitting the equity term in Equation 1 into two components, we find that this effect is confined to firm-specific returns, and is not visible in market-wide (common-factor) returns.

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8 An overview of this model with links to further details may be found at http://www.barra.com/support/equity_models/#us.
Application to Risk Modeling

Traditional models of risk for corporate bonds are based on the attribution of bond returns to interest rate and residual credit spread factors. Interest rate factor returns, derived from either returns to government bonds or changes in swap rates, are exogenous to the corporate bond universe. Spread factors are derived by categorizing credit-risky bonds as, e.g., AA-rated industrials, A-rated financials, and so on, then regressing a set of common spreads on bond returns residual to interest rates. The spread factors are often referred to as credit risk factors, although they are not directly due to credit risk in the sense of issuer-level risk due to downgrade or default. Rather, they account for market-wide repricing of credit risk for groups of similar issuers. Historical time series of these factor innovations (“returns”) are then used to construct forecasts of the future distribution of returns.

This attribution works quite well for high-grade corporate bonds, for which interest rate changes are the dominant source of cross-sectional return variation, with common spread factors accounting for a quarter to half of the remaining return variance (see Figure 6). However, as we move down the credit quality spectrum, interest rates and common spreads account for a rapidly decreasing share of return. Changes in government bond yields or swap rates and common spread factors tell us very little about returns to bonds rated BB and below. After incorporating spread factors, the $R^2$ of the model ranges from below 40% for bonds rated BB and B to below 20% for bonds rated CCC.

In order to obtain a model for application to risk forecasting, we will have to estimate parameters of curves qualitatively similar to those of Figure 1. The empirical behavior shown in Figure 2 is reasonably well captured by the functions

Equation 3a
$$\beta_{E_k} = a_1 \left(1 - \exp\left(-\left(a_2 + a_3 D\right) \cdot \text{OAS}\right)\right) a_6$$

Equation 3b
$$\beta_{R_k} = a_1 \left(1 - \exp\left(-a_2 \cdot \text{OAS}\right)\right) a_6$$

where $a_1, \ldots, a_6$ are parameters defining the shapes. The choice of these functional forms is somewhat arbitrary, dictated primarily by the desire that the functions be asymptotically constant at large OAS, lie between 0 and 1, and be described by a small number of parameters, to avoid overfitting. We require fitted functional forms such as these, rather than simply using the results of, say, binned regressions, for two reasons. One is that we expect the true relationship between bond and equity returns to vary smoothly and monotonically with credit quality, whereas binned regressions would give us discontinuous and possibly non-monotonic behavior (due to statistical errors). Second, at extreme values (low or high) of the OAS, we have relatively little data for estimation of $\beta_{R_k}$ and $\beta_{E_k}$, so the estimates will be quite noisy. By imposing functional forms with reasonable limiting behavior, we can obtain usable estimates for the values in these extremes.

We are also free to adapt the heuristic functional forms of $\beta_{R_k}$ and $\beta_{E_k}$ and to accommodate dependence on additional characteristics of bonds or firms. For example, we found that $\beta_{E_k}$ depends on duration as well as OAS, and therefore in application we modify Equation 3b to

Equation 3b'
$$\beta_{E_k} = a_1 \left(1 - \exp\left(-\left(a_2 + a_3 D\right) \cdot \text{OAS}\right)\right) a_6$$

where $D$ is the bond’s duration.

Figure 3 shows the functions $\beta_{R_k}$ and $\beta_{E_k}$ of Equations 3a, b with parameters estimated by GLS from the same data as the binned regressions of Figure 2, also included for comparison.
The simple functional forms manifestly provide reasonable smooth interpolation and extrapolation of the empirical bond return relationships. A simple application of the heuristic representation for is the forecasting of the distribution of bond prices or spreads. Holding all but the equity contribution fixed, for an individual bond, Equation 1 may be integrated to give

\[
\int_{B_i}^{B_f} \frac{dB}{B \beta_E(s(B))} = \ln(1+r_E)
\]

where \(B_i\) and \(B_f\) are the starting and ending bond prices and \(s(B)\) is the OAS as a function of bond price. With the further approximation that the spread duration, \(D_E = \left( -\frac{1}{B \frac{dB}{ds}} \right)\), is approximately independent of bond price, we obtain the simplification

\[
\int_{s_{B_i}}^{s_{B_f}} \frac{ds}{\beta_E(s)} = \frac{-1}{D_E} \ln(1+r_E)
\]

Given a model for the distribution of the equity return \(r_E\) on the right hand side of this equation (for example, normal), we obtain a distribution for the bond spread. Some examples are shown in Figure 4, based on volatility forecasts and initial OAS’s for bonds as of June 2002.

A related application of Equation 5 is scenario analysis. Instead of looking at a forecast of the distribution of \(r_E\), we can look at bond spreads conditional on particular scenarios for \(r_E\). For example, we can ask what would happen to

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Footnote: Generically, Equation 1 describes a path-dependent relationship between equity and interest rate changes and the bond return, because the \(\beta\) functions are not components of an exact differential. This complication goes away if we restrict attention to bond returns implied by equity returns alone.
bond spreads if the issuer suffers a two standard deviation drop in equity value (whose magnitude could be based on forecasts from our equity risk model). An example of this analysis, which we refer to as Equity Return Implied Spread, or ERIS, is shown in Figure 5 for Ford based on data as of 12/31/2002.

For a portfolio manager, the probability curves of Figure 4 leave out important information, namely the correlations in the outcomes for different bonds. The issuers of these bonds include an airline, a chemical producer, a consumer credit firm, two telecoms and a packaging manufacturer. We expect that the positive and negative outcomes in Figure 4 are neither perfectly correlated nor completely independent. One approach to predicting these correlations is through a factor model of returns.

The factor-based approach to risk modeling mentioned earlier for bonds was first developed by Barra in the 1970s for application to equities and later to bonds, and widely adopted since then. The model decomposes asset returns into components attributable to common factors and residuals, and constructs forecasts for the factor return distributions, or at least their second moments (variances and covariances). Such models are extremely useful for portfolio risk forecasting, as they permit the aggregation of risk forecasts for individual assets to the portfolio.

As already noted, the standard “interest rates plus spreads” factor model for bonds works quite well for higher quality investment grade bonds, but not very well for low-grade bonds, primarily because changes in default-free interest rates are not useful explanatory factors for such bonds, and sector/rating spreads alone don’t contribute a great deal to explaining their returns.

On the other hand, as we’ve seen, returns to low-grade bonds are explained quite well by equity returns. Combining an interest rate factor model with an equity factor model, and using Equations 1 and 3a and b or b', we obtain a model of the common factors and exposures to which bonds are exposed across the spectrum of credit quality.

That is, a bond with OAS is exposed to interest rate factors with exposures given by it multipliers by the computed interest rate factor exposures, and to equity factors with exposures multiplied by the factor exposures of the equity.

As shown in Figure 2, the explanatory power of this model falls to a low level of $R^2 = 0.14$ at boundary between investment and high-yield bonds. Motivated by earlier academic studies and by our observations that the sector/rating spread factors contribute roughly 0.1 to 0.2 of improvement in $R^2$ for intermediate to lower-grade bonds, we extend the model by looking for additional factors to explain the residuals from Equation 1.

A simple model (though perhaps having too many factors) is a sector/rating spread model of the residuals. The residuals of Equation 1 are regressed on a spread model with each bond exposed to one spread factor, based on sector and rating, with magnitude equal to minus the...
spread duration. This is very similar to the method used to estimate the “interest rates plus spreads” model currently delivered by Barra, except that rather than fitting the spread model to the residuals after accounting for interest rates alone (with unscaled exposures), the spread model is now applied to the residuals after accounting both for interest rates (with \( \beta_{\text{IR}} \)-scaled exposures) and equity returns. The resulting model of returns can be written as

\[
\Delta \tau^i_s = \beta_{\text{IR}} \tau_{s,m}^i + \beta_E \tau_{s,e}^i + (-D^s) \Delta s^t_{\text{CIS}} + \eta_B^i
\]

where \( \Delta s^t_{\text{CIS}} \) is the “spread” change for the issuer’s sector \( C_{\text{IS}} \) and \( \eta_B^i \) is the remaining residual return. Note that the spread changes \( \Delta s^t_{\text{CIS}} \) are not changes to market spreads for bonds in each sector: first because a portion of each bond’s spread change has already been accounted for by the equity return, and second because the interest rate exposures are scaled by \( \beta_{\text{IR}} < 1 \). We refer to them as spread changes because the bonds’ exposures to them are equal to their spread durations, and because for high-grade bonds, for which \( \beta_{\text{IR}} \sim 1 \) and \( \beta_{\text{IR}} \ll 1 \), they are very close to the sector/rating spread change.

The added explanatory \( R^2 \) produced by this extension is shown in Figure 6, comparing the \( R^2 \) of the current Barra model, a model of bond returns based on interest rates only, the current Barra rates + spreads model of (Equation 2), and the two equity-linked models, Equation 1, and Equation 6. The calculations are based on the regression formulas using the heuristic functional forms of Equations 3a and 3b’. In each case, bonds have been grouped both by OAS bin (solid lines) and by coarse agency rating (data points). The sharp dip of Figure 2 at around 400 basis points persists, though it is less pronounced in the agency rating bins.

Further insight into the difference between the rates-plus-spreads model and the one based on Equation 6 is evident from Figure 7, which shows the first three eigenvectors of the spread covariance matrix from both models. The eigenvectors have been scaled by their volatilities to show the contribution of each factor to residual spread volatility. With the single exception of the transport-B spread, the scaled eigenvectors of ECR residual spreads are substantially smaller than those of the rates-plus-spreads model. This is just a consequence of the fact that the equity return accounts for an appreciable component of return that is instead accounted for by spreads in the rates-plus-spreads framework.
Summary

We have identified a missing ingredient in explaining returns of moderate to low credit quality bonds. A significant component of their return is explained by changes in firm values as seen in the issuers’ equity returns. Indeed, for lower credit quality bonds, equity returns explain much more of bond returns than do changes in default-free interest rates or market-wide spreads.

Based on this analysis, we can construct heuristic estimates for the exposure of a bond to equity market risk factors and equity issuer-specific risk. We expect the empirical credit risk model to provide a significant improvement in the quality of risk forecasts over more traditional bond risk models.

Figure 7

First three eigenvectors scaled by their volatilities for the spread covariance matrices of the rates + spreads model (upper figure) and from Equation 6 (lower figure). With the exception of B-rated transports, the magnitudes are appreciably smaller in the lower figure than in the upper. The large correlation between B-rated transports and utilities evident in the upper figure also disappears in the lower. The persistent large transport-B spread factor volatility is attributable to the 9/11/01 attacks.
The following discussion is excerpted from the white paper “Forecasting US Equity Risk Over Different Horizons”. The complete document is available on Barra’s client support website: http://www.barra.com/support/

Dynamic Volatility, Investment Horizon, and Risk Forecasting

US equity risk has always been dynamic, displaying irregular variations from month to month and year to year. Figure 1 shows the history of realized monthly volatility in the S&P 500 Index from 1980 to 2004; the estimates of volatility are based on daily returns. The crash of October 1987 stands out as an isolated feature—but there are other important trends to note. Risk moves up and down moderately throughout the 1980s, followed by an unusually quiet period in the mid-1990s. With the onset of the Internet bubble in the late 1990s, risk levels rise rapidly and remain at remarkably high levels for years—in contrast with the earlier periods, in which such high levels are never sustained for more than a few months.

Barra’s USE3 model was released in 1997, shortly before the volatility explosion of the late 1990s. Its forecasts are based on a detailed cross-sectional analysis of monthly asset-level returns. The analysis decomposes the return to an individual asset into a specific return that is statistically independent of the returns to other assets, and a return from common factors that are shared with many other assets. USE3 forecasts of common factor risk depend on the assumption that the best forecast of future risk is based on a multi-year historical average.

By 2000, this assumption of weakly dynamic risk had been violated severely for some of the common factors. With the benefit of hindsight, it also became clear that similar variations in factor risk levels had always been present, although they had never asserted themselves with such violence before. Strongly dynamic volatility complicates the task of forecasting risk, since future risk will differ from past risk,
sometimes presenting behavior that appears entirely new. The past still provides our guide to the future, but it must be employed judiciously. The question becomes, exactly what range of the past should be used and how?

How a risk forecast should respond to past events depends, at least to some extent, on the investor who uses it. The investment horizon, defined as the typical lifetime of active portfolio positions or the average time between major rebalances, is particularly relevant. Figure 2 shows realized risk of the S&P500 over one-month and six-month horizons (the estimates of historical volatility are made from daily returns data, as in Figure 1).

Risk levels over the shorter horizon are far more volatile than levels over the longer horizon. The impact of the investment horizon increases with the variability of risk: if risk levels don’t change from month to month, risk is the same for any investment horizon. This observation is hardly surprising, but it is important. Risk forecasts aimed at investors with shorter horizons should naturally be more volatile than risk forecasts aimed at longer-term investors. The short-horizon forecast must respond more aggressively to rapid—and typically short-lived—changes in risk levels than its long-horizon counterpart.

Thus, a short-horizon forecast will emphasize the very recent past more strongly than a long-term forecast.

Figure 2 also suggests a second point. Since risk over monthly horizons is both volatile and erratic, it will almost certainly be impossible to predict each peak and valley. Rather, a good risk forecast for the coming month will aim at an expected average level. It will be responsive, but still smoother than the realized volatility itself. Thus a good forecast for monthly risk should be acceptable over horizons from one to several months. At sufficiently long investment horizons, realized risk will vary so gradually that users will find the near-term forecast unnecessarily volatile, and will prefer a smoother version.

New Model, Two Versions

The Multiple-Horizon Equity Model was created to address these dynamic aspects of risk. The new model comes in two versions, Long-Term (USE3L) and Short-Term (USE3S). Both base their forecasts on daily returns, and both respond more sensitively to changing conditions. The more responsive is the Short-Term model, designed to forecast risk over one to six-month horizons. Its counterpart, the Long-Term model, is intended for users with semi-annual or longer horizons. Both models employ a common architecture, described below.

Forecasting Methodology

Common Factor Risk

The factor structure of USE3 provides an excellent basis for the cross-sectional modeling of common return over horizons varying from a month to a year. The Multiple-Horizon Equity Model adopts the factor structure without
modification. In doing so, we preserve USE3’s cross-sectional detail while striving to refine its temporal resolution and produce more responsive risk forecasts.

To respond quickly to changing risk levels, risk forecasts must emphasize more recent data, which effectively shortens the data history. In

**Strongly dynamic volatility complicates the task of forecasting risk, since future risk will differ from past risk... The question becomes, exactly what range of the past should be used and how?**

EWMA (exponentially weighted moving average) forecasts, the half-life specifies how the weights placed on historical observations decline through time, with more recent observations receiving greater weight. For example, a half-life of 12 months implies that observations from 12 months ago receive half the weight of the current month’s observations. One way of achieving greater model responsiveness is therefore to retain monthly returns data, and simply to shorten the half-life to 12 or 24 months in constructing factor covariance matrices. The disadvantage of such short half-lives is that they reduce the effective sample size significantly, resulting in noisy and less precise forecasts.

Thus, although using monthly factor returns with a short half-life would increase responsiveness, it is a poor modeling choice. Higher frequency data allows us to shorten the half-life while maintaining a large enough sample of historical observations that precision is unimpaired. Since there are roughly 20 trading days per month, daily data present a natural solution to the data density problem.

The Multiple-Horizon Equity Model relies on a time series of daily factor returns in constructing the factor covariance matrix. These factor returns are obtained by regressing daily asset returns against factor exposures that were computed at the start of each month. The regression is identical in all other respects to USE3’s monthly factor return regression.

Some care is required in aggregating risk estimates from daily data to horizons of a month or longer. Daily returns are more susceptible than monthly returns to the influences of market microstructure (asynchronous trading, bid-ask spreads, limits on daily price movements, etc.) and other evanescent market phenomena (information diffusion effects, under- and over-reaction). These effects can induce lead-lag relationships in asset-level returns, which then manifest themselves as serial correlations in factor returns.

The effect of serial correlations is intuitively easy to understand.¹ If positive (negative) returns on one day are likely to be followed by positive (negative) returns on the next day, the returns reinforce one another and monthly volatility is greater than if the returns on each day were independent. If the returns on one day tend to be cancelled by subsequent returns, monthly volatility is subdued. For example, Momentum is one of the factors most dramatically affected by serial correlation. The monthly variance of the Momentum factor tends to be magnified by positive serial correlations extending over several days. Although the enhancement can vary with time, throughout the period from 1997 to the present serial correlations have consistently increased the monthly variance of Momentum returns to almost twice that of the value obtained by simply scaling the daily variance.

¹ For a complete technical account of how serial correlations enter the risk forecasts of USE3S and USE3L, see Appendix of the complete white paper “Forecasting US Equity Risk over Different Horizons” at http://www.barra.com/support.
To achieve a covariance forecast that is both responsive and statistically sound, we separate the variance portion from the correlation portion of the covariance matrix. Correlations between factors tend to be relatively stable over time: the economic reason governing that relationship is not likely to disappear overnight. As a result, we use a longer half-life to estimate factor serial and cross-correlations. Factor variances, which can change quite rapidly, are estimated using a shorter half-life to achieve the desired level of model responsiveness. Factor covariance forecasts in the new model thus depend on three parameters: a serial correlation parameter $N$ that indicates the maximum number of subsequent days over which non-negligible serial correlations are assumed to extend, a relatively long correlation half-life, and a shorter variance half life.

**Specific Risk**

In contrast to common factor risk, monthly returns data are quite sufficient to accurately estimate the specific risk level realized in a given month because the large cross-sectional sample of returns effectively provides many independent samples of the distribution. Figure 3 illustrates the responsiveness and accuracy of the USE3 specific risk forecast.

Although we experimented with modifications to the existing specific risk model, including the use of daily returns, alternative models could not consistently improve on its performance. We therefore adopted the USE3 specific risk forecasting system for both the Short-Term model and the Long-Term model without modification.

**Modeling Short-Term Risk: Model Characteristics and Evaluation**

The forecasting parameters of the Short-Term model were selected from a large set of possibilities to produce the most consistently accurate risk forecasts over a wide range of market conditions. Table 1 describes the parameters of the Short-Term model.

<table>
<thead>
<tr>
<th>Properties of the Short-Term Model (USE3S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-life for Variances (days)</td>
</tr>
<tr>
<td>Half-life for Correlations (days)</td>
</tr>
<tr>
<td>Serial Correlation Length $N$</td>
</tr>
</tbody>
</table>

Note: 480 days corresponds to approximately two years of trading days and 90 days to about four and a half months.

Figures 4a and 4b (on the following page) compare forecast and realized volatility for the Earnings Yield style factor and the Computer Hardware industry factor.

The graphs show that the risk forecasts of the Short-Term model track realized volatility levels closely over most time periods. The most severe discrepancy between forecast and realization occurs when the volatility of the Earnings Yield factor rises by nearly a factor of 3 and then plunges downward again. Even in this case the forecast follows the changing level aggressively, trailing realized levels by only a few months. It achieves this without the enormous month-to-month fluctuations seen in the realized history. Since these fluctuations are at least partly due to uncertainty in estimation (recall that each estimate of realized monthly risk comes from
about 20 daily returns), monthly risk level changes are difficult to accurately measure, let alone forecast. A good forecast aims at a smoother expected average level, and this is what the Short-Term model provides.

Table 2 compares bias statistics for USE3 and the Short-Term model (USE3S) for a broad selection of market and active portfolios. The bias statistic is an intuitive and commonly used indicator of forecast accuracy. It is the ratio of realized risk to forecast risk, averaged over a period of time. An accurate risk forecast will yield a bias statistic close to one. If the bias statistic lies significantly below one then the model has over-predicted risk. Conversely, if the bias statistic is significantly above one, then the model has under-predicted risk. In order to examine model performance in periods with wildly differing behaviors, we present performance results across three different time periods.

USE3 predicts short-term risk with reasonable accuracy over most of the 1990s; in this period forecast responsiveness is not a primary determinant of performance, and the model does well. The latter period shows the stress produced by a large and prolonged increase in risk. The Short-Term model, with a much shorter half-life, significantly out-performs USE3 in this more volatile period. The Short-Term model successfully forecasts total and active risk throughout the test history, indicating that the model has accurately captured relations between different sources of risk.

Modeling Long-Term Risk: Model Characteristics and Evaluation

As the risk horizon lengthens, month-to-month risk variability decreases. A model that works well over 3-month horizons, for example, may well prove too volatile for users longer horizons — responsiveness for the near-term becomes overreaction for longer terms. Unnecessary forecast variability can impair a strategy’s performance and stimulate needless and costly

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2 For active portfolios, the return tested is the portfolio return minus the return to a benchmark portfolio, named in parentheses.
3 For a quantitative discussion of the construction and use of bias statistics, see Appendix B of the complete white paper “Forecasting US Equity Risk Over Different Horizons.”
4 The word “significantly” is important here. Like any other quantity obtained through statistical measurement, there is some uncertainty in the bias statistic. This uncertainty generally decreases as the size of the sample used for estimation increases; that is, as the time over which performance is averaged lengthens.
5 Realized volatility is standard deviation of daily returns over each month adjusted for first-order serial correlation.
6 For additional bias statistics on factor portfolios, please see the complete white paper “Forecasting US Equity Risk Over Different Horizons.”
trading. Long-term investors require a model that suppresses rapid forecast fluctuations while preserving the ability to follow developing trends.

A wide range of candidate factor covariance forecasting schemes was reviewed for the Long-Term model. Table 3 shows the parameters selected. The Long-Term model adopts longer estimation half-lives to achieve an attractive balance between forecast responsiveness, accuracy, and stability.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>USE3S</th>
<th>USE3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1993</td>
<td>1998</td>
</tr>
<tr>
<td></td>
<td>1993</td>
<td>1998</td>
</tr>
<tr>
<td>S&amp;P 100</td>
<td>0.88</td>
<td>1.06</td>
</tr>
<tr>
<td>S&amp;P 500</td>
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<td>1.06</td>
</tr>
<tr>
<td>S&amp;P 500/Barra Growth</td>
<td>0.95</td>
<td>1.04</td>
</tr>
<tr>
<td>S&amp;P 500/Barra Value</td>
<td>0.92</td>
<td>1.02</td>
</tr>
<tr>
<td>S&amp;P/Barra MidCap 400</td>
<td>—</td>
<td>1.00</td>
</tr>
<tr>
<td>S&amp;P/Barra MidCap 400 Growth</td>
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<td>1.02</td>
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<tr>
<td>S&amp;P/Barra MidCap 400 Value</td>
<td>—</td>
<td>1.00</td>
</tr>
<tr>
<td>S&amp;P/Barra SmallCap 600</td>
<td>—</td>
<td>1.05</td>
</tr>
<tr>
<td>S&amp;P/Barra SmallCap 600 Growth</td>
<td>—</td>
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<td>S&amp;P/Barra SmallCap 600 Value</td>
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<td>1.05</td>
</tr>
<tr>
<td>Top 50 by Capitalization</td>
<td>0.91</td>
<td>0.97</td>
</tr>
<tr>
<td>Random Portfolio</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>S&amp;P 500/Barra Growth/Value vs. S&amp;P 500</td>
<td>1.26*</td>
<td>1.01</td>
</tr>
<tr>
<td>S&amp;P/Barra MidCap 400 Growth/Value vs. S&amp;P MidCap 400</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>S&amp;P/Barra SmallCap 600 Growth/Value vs. S&amp;P SmallCap 600</td>
<td>—</td>
<td>1.12</td>
</tr>
<tr>
<td>Random Portfolio vs. Estimation Universe</td>
<td>1.07</td>
<td>1.05</td>
</tr>
</tbody>
</table>

* Bias statistic is significantly different from 1.
** When forming bias statistics, the extreme events of Oct 1987 and Aug 1998 are excluded from the sample.
Blank entries exist where the portfolio does not exist for most of the period.

Table 3
Properties of the Short-Term Model (USE3S)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-life for Variances (days)</td>
<td>250</td>
</tr>
<tr>
<td>Half-life for Correlations (days)</td>
<td>750</td>
</tr>
<tr>
<td>Serial Correlation Length N</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: 250 days corresponds to approximately one year of trading days.

Model Evaluation

Figures 5a and 5b illustrate the behavior of the Long-Term Model. As points of reference, forecasts from the Short-Term model and USE3 are also plotted. Realized long-term volatility is represented by a forward-looking 6-month average of daily factor volatility, corrected for serial correlation. It is important to note that when the realized volatility level flies upward, it does so 6 months before the actual volatility appears in returns. As such, it is a prescient ideal target for real risk models, which can only base their forecasts on events that already have taken place. Despite this challenge, the figures show that the Long-Term model’s forecasts track 6-month realized volatility levels closely overall—although perhaps not as tenaciously
as those of the Short-Term model. As intended, the Long-Term model responds well to prolonged excursions in risk levels while “averaging over” shorter-lived fluctuations. The result is that the Long-Term model’s forecasts adjust more conservatively and gradually than forecasts produced by the Short-term model. In contrast, USE3 forecasts express very long-term averages, and typically change by only small amounts. As illustrated by Figure 5a for the Momentum style factor, when extremely violent events do change the forecasts of USE3, these changes remain imprinted in subsequent forecasts for many years.

Table 4 (on the following page) lists the bias statistics of a selection of market and active portfolios in two historical intervals. Since the relevant investment horizons are longer than in the case of USE3S, the statistics are calculated from returns over 6-month intervals. The number of independent six-month intervals later than 1997 is relatively small, and so the sample is split equally into two nine-year periods: 1986–1994 and 1995–2003. To prevent the choice of starting dates from influencing the results, each month in a period was used as a starting point for a 6-month return, so that the returns intervals overlap.

The models perform similarly in the first period, an unsurprising result given the risk level variations during this time are generally not too severe. In the second period, its shorter half-life allows the Long-Term model to perform significantly better.

In particular, the challenge to risk forecasts posed by dynamic volatility appears vividly within the 1995–2003 period biases for active returns to Growth and Value portfolios. These results underscore the importance of responsiveness. When volatility changes rapidly, models succeed or fail according to their ability to react. The Long-term model’s forecasts are based on averages of recent data, and benefit enormously from the high density of daily data.

Investment Horizon and Model Choice

Which investors should use the Short-Term model, and which the Long-Term model? While both vary their forecasts to follow changing risk levels, the Short-Term model tracks rapid variations more closely than its long-term counterpart. The Long-Term model assumes this fore-

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For active portfolios, the return tested is the portfolio return minus the return to a benchmark portfolio, named in parentheses.
Barra Events
& Industry Conferences

JUNE — AUGUST 2004
JUNE

2  Aegis Portfolio Management Workshop  
   Boston, Massachusetts

9  Introducing the New Multiple-Horizon US Equity Model  
   San Francisco, California

10 Introducing the New Multiple-Horizon US Equity Model  
   Los Angeles, California

15 Introducing the New Multiple-Horizon US Equity Model  
   London, United Kingdom

16 Aegis Portfolio Management Workshop  
   London, United Kingdom

17 Cosmos Risk, Scenario and Portfolio Construction Workshop  
   London, United Kingdom

23 Introducing the New Multiple-Horizon US Equity Model  
   Chicago, Illinois

23 International Research Conference  
   Cape Town, South Africa

24 International Research Conference  
   Johannesburg, South Africa

24 Aegis Portfolio Management Workshop  
   Frankfurt, Germany

24 Introducing the New Multiple-Horizon US Equity Model  
   Toronto, Ontario, Canada

JULY

8 Introducing the New Multiple-Horizon US Equity Model  
   Online

13 Aegis Portfolio Management Workshop  
   London, United Kingdom

21 Aegis Portfolio Management Workshop  
   New York, New York

22 Out-Perform Your Benchmark with Equity Risk Implied Spreads  
   Online

AUGUST

8 Aegis Portfolio Management Workshop  
   San Francisco, California

Barra Speaking Engagements

June 21–22  
   Asset Allocation Summit  
   London, United Kingdom  
   Barra Speaker: Tom Koundakjian

June 22–23  
   Risk USA  
   Boston, Massachusetts  
   Barra Speakers: Tim Backshall and Guy Miller

June 15–17  
   Global Borrowers and Investors Forum  
   London, United Kingdom  
   Barra Speaker: Tim Backshall

July 6–9  
   14th Annual Fund Forum International  
   Monaco  
   Barra Speaker: Andrew Rudd
Barra Seminars

Barra seminars, which are open to both clients and prospective clients, provide an excellent vehicle for learning about Barra’s research and how it applies to events currently shaping the investment landscape.

Introducing the New Multiple-Horizon US Equity Model

<table>
<thead>
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<tbody>
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<td>Toronto, Ontario, Canada</td>
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<td>June 15</td>
<td>London, United Kingdom</td>
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<tr>
<td>July 8</td>
<td>Online</td>
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</tbody>
</table>

The Multiple-Horizon Equity Model applies daily returns to the proven framework of Barra’s US model to achieve more responsive and accurate risk forecasts in today’s dynamic markets. This half-day session examines our research and reviews the construction of both the Short-Term and Long-Term versions of the new Multiple-Horizon US Equity Model.

Out-Perform Your Benchmark with Equity Risk Implied Spreads

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<tr>
<th>Date</th>
<th>Location</th>
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<tbody>
<tr>
<td>July 22</td>
<td>Online</td>
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</tbody>
</table>

Although theoretically capable of linking bond returns and default probabilities, traditional credit risk models have not had great practical success. By using an empirical model of the relationship between the return to a credit-risky bond and its issuer’s equity, this new approach sidesteps these challenges and provides new insight into the sources of corporate bond returns.

Barra Client Education

Aegis Portfolio Management Workshop

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Cosmos Risk, Scenario, and Portfolio Construction Workshop

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<th>Date</th>
<th>Location</th>
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</thead>
<tbody>
<tr>
<td>June 17</td>
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</table>

Online Education

In addition to the hands-on product workshops listed here, Barra also provides numerous online educational opportunities for its clients. Learning tools include self-paced workshops, interactive tutorials, and pre-recorded presentations. Please visit the Client Support section at www.barra.com to learn more.
Barra Research Events

Barra’s Research Events provide a forum for presenting our latest research in risk modeling and portfolio management. Conducted at a high level of quantitative rigor, these events typically include noted academics and practitioners as guest speakers, in addition to the renowned Barra research staff.

19th Annual Barra International Research Conference: Mastering Risk Management in the Front Office

June 23 | Cape Town, South Africa
June 24 | Johannesburg, South Africa

This conference will focus on the latest trends in risk management, investment performance measurement and attribution. Topics will include risk budgeting and the integration of risk management in the portfolio management process. These concepts will be explored with special reference to the management of balanced funds. Several case studies analysing the risk and performance of South African balanced unit trusts will be presented. In addition, Reuters will be present to provide further insight into these topics.

Industry Conferences

Look for Barra’s booth and speakers at these events:

Asset Allocation Summit

June 21–22 | London, United Kingdom
Sponsored By: IRC Conferences
Barra Speaker: Tom Koundakjian
Millennium Gloucester Hotel
London, United Kingdom

Risk USA

June 22–23 | Boston, Massachusetts
Sponsored By: Risk Waters Group
Barra Speakers: Tim Backshall and Guy Miller
Seaport Hotel & World Trade Center
Boston, Massachusetts

Contact Information

For more information or to register for any of the Barra seminars or workshops listed, please visit our web site at www.barra.com

Save the Date

September 19–22
Equity Portfolio Management Best Practices Conference
(formerly known as Equity Portfolio Management Seminar)
Chicago, Illinois

October 7–8
2nd Annual Total Risk User Summit
Amsterdam, The Netherlands
cast variability to be unnecessarily severe for long-term investors, and smoothes its forecasts. The choice of one model as most appropriate depends on these differences and their implications.

Variability in a risk forecast can drive undesirable turnover. Based on our tests of model performance at different horizons and the impact of forecast variability on portfolio turnover, a rough guide to model choice emerges. For strategies with horizons of 1–6 months, the Short-Term model is a good choice. Managers with horizons longer than a half year generally will prefer the Long-Term model. Near the interface between these extremes, the decision will depend on how the individual situation dictates an acceptable balance between responsiveness and stability.

### Conclusion

Daily returns data enable Barra’s new Multiple-Horizon Equity Model to capture changing risk levels promptly and sensitively. Its increased responsiveness makes it possible to distinguish between the differing needs of investors with different risk horizons. The short-term version, USE3S, offers aggressive risk-level tracking; its high accuracy makes it a natural choice for use in strategies with horizons of one to six months. Users with longer horizons will derive less benefit from assiduously following risk fluctuations on such short timescales; the long-term version, USE3L, presents them with more smoothly varying risk forecasts that respond more conservatively to events from the past few months.
In our last issue of Horizon (January 2004), we published two of three articles in a series that focuses on the benefits of the equity portion of the Barra Integrated Model (BIM).

The first article, “The Barra Integrated Model: A Breakthrough in Modeling Global Equity,” explained how and why BIM makes a difference in analyzing global equity risk.

The case study, “Analyzing Risk with the Barra Integrated Model” demonstrated how a global investment management firm can integrate the BIM into its investment process.

In the following article, we discuss the practical applications of the equity portion of the Barra Integrated Model and review the business implications of this new model for investment managers.

Introduction

The Barra Integrated Model is a global multi-asset class model that forecasts asset and portfolio level risk for global equities, bonds and currencies. Using innovative methods to couple broad asset coverage with detailed local market models, BIM provides in-depth analyses for single-country and global portfolios across asset classes.

The Barra Integrated Model for equities is a major innovation that brings together more than 40 customized single market models in a unified global framework. Each locally estimated model captures the unique industries, style influences and idiosyncratic nature of that market. BIM, the union of these detailed single market models, provides a comprehensive global analysis that incorporates these purely local sources of risk and return. In contrast, classical global equity models reflect local influences largely through a single local market or country factor.

There are a number of ways in which BIM can enhance the investment process and portfolio results. First, the bottom-up construction and local depth provided by BIM results in significantly more accurate estimates of tracking error and its decomposition. This additional level of granularity allows for precise control over portfolio construction and diversification, and allows portfolio managers to deliver more consistent results. Second, BIM’s unified framework allows regional portfolio managers, risk officers and chief investment officers to rely on a single model for portfolio construction, risk management and enterprise risk analysis throughout the organization, yielding a common language and framework invaluable for
global asset management firms. Finally, the many local factors identified in the Barra Integrated Model reveal new sources of return that can be exploited in active investment strategies to enhance risk-adjusted returns.

Enhanced Accuracy in the Forecast and Decomposition of Risk

Multiple factor models are based on the idea that there are sources of risk and return that are common to many securities. For example, large cap stocks tend to move together, financial stocks tend to move together, and so on. These common sources of risk or “common factors” are loosely grouped as industries (such as Banks or Construction) and styles (such as Size, Value, and Momentum).

Classic global models define common factors at the global level and therefore by construction misallocate a large portion of purely local factor returns as security specific returns, leaving some local sources unrecognized. Instead of making global generalizations, BIM’s bottom-up construction allows us to define common factors at the local level. This ability to capture purely local common factors allows us to differentiate global risks from purely local risks, which significantly improves the accuracy of predicted tracking error. The resulting decomposition of risk is more granular, and each piece is a more accurate and actionable description of risk.

This is best illustrated through a simple but telling example: Consider an active portfolio of US and UK stocks. Like its benchmark, the sample portfolio is comprised of 50% Bank securities and 50% Business and Public Services securities. Both the portfolio and the benchmark hold 50% US stocks and 50% UK stocks. The portfolio has a 15% overweight in UK Banks and US Business Services and a 15% underweight in UK Business Services and US Banks relative to the benchmark. However, under a classical global framework, this portfolio carries no active industry or country exposures.

Table 1 describes the active exposures of our sample portfolio using Barra’s Global Equity Model (GEM).

Under a classical global framework, the portfolio appears to be closely tracking its benchmark with no active country, currency or industry exposure.

<table>
<thead>
<tr>
<th>Risk Source</th>
<th>Managed</th>
<th>Benchmark</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Style Exposures</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>-0.31</td>
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<tr>
<td>Success</td>
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<td>0.13</td>
<td>-0.07</td>
</tr>
<tr>
<td>Value</td>
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<td>0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>Variability in Markets</td>
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<td>-0.11</td>
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<td><strong>Country Exposures</strong></td>
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<tr>
<td>Business &amp; Public Services</td>
<td>50%</td>
<td>50%</td>
<td>0%</td>
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<tr>
<td>Banking</td>
<td>50%</td>
<td>50%</td>
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<tr>
<td><strong>Industry Exposures</strong></td>
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<td>Business &amp; Public Services</td>
<td>50%</td>
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<tr>
<td>Banking</td>
<td>50%</td>
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<tr>
<td><strong>Currency Exposures</strong></td>
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<tr>
<td>UK</td>
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<td>50%</td>
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<tr>
<td>US</td>
<td>50%</td>
<td>50%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note that Style biases are small, and that GEM does not recognize any active industry exposure.
weights. Style biases are small, with a moderate overweight to Size (the benchmark is weighted towards small-cap stocks). A portfolio manager reviewing these results would believe the portfolio contained little active risk.

Analyzing the same portfolio with BIM yields different—and more enlightening—results. The local US and UK models have more granular industry and style factors, selected to capture the unique attributes of each market. For example, instead of a single, global Bank industry, BIM classifies bank related securities in several more specific local industries, including US Banks, US Thrifts, US Securities Management, UK Banks and so on. Table 2 and Figure 1 give the local common factor exposures for the portfolio for industry and style factors, respectively. For simplicity, Table 2 groups BIM’s local industries by GICS classification.

<table>
<thead>
<tr>
<th>GICS Industry</th>
<th>UK (%)</th>
<th>US (%)</th>
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<tbody>
<tr>
<td>Commercial Banks</td>
<td>14.70</td>
<td>-10.60</td>
</tr>
<tr>
<td>Thrifts &amp; Mortgage Finance</td>
<td>0.30</td>
<td>-3.30</td>
</tr>
<tr>
<td>Capital Markets</td>
<td>N/A</td>
<td>-1.10</td>
</tr>
<tr>
<td>Internet Software &amp; Services</td>
<td>-0.15</td>
<td>0.61</td>
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<tr>
<td>Media</td>
<td>-2.59</td>
<td>0.63</td>
</tr>
<tr>
<td>Air Freight &amp; Logistics</td>
<td>-0.74</td>
<td>1.10</td>
</tr>
<tr>
<td>Commercial Services &amp; Supplies</td>
<td>-4.97</td>
<td>1.76</td>
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<tr>
<td>Computers &amp; Peripherals</td>
<td>-0.03</td>
<td>1.77</td>
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<td>IT Services</td>
<td>-1.20</td>
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<tr>
<td>Health Care Providers &amp; Services</td>
<td>-0.23</td>
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<tr>
<td>Software</td>
<td>-1.50</td>
<td>5.21</td>
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<tr>
<td>Transportation Infrastructure</td>
<td>-3.60</td>
<td>N/A</td>
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</table>

BIM’s more detailed framework reveals that the sample portfolio clearly has some significant industry biases. Most notably, the portfolio has a large overweight to UK Commercial Banks, and a large under-weight to US Commercial Banks. These biases can clearly have a meaningful impact on portfolio performance. For example, should banks perform well in the US but poorly in the UK, then the portfolio’s supposedly neutral bias to banks would doubly detract from portfolio performance. A global industry framework does not allow portfolio managers to monitor and control this kind of local industry bias.

Similarly, local style biases can have a material impact on returns. Figure 1 describes the local active style exposures described by BIM. While a global style framework suggests the portfolio has a global overweight to larger stocks, BIM reveals that the portfolio has an insignificant Size bias in the US, but a large bias to large-cap stocks in the UK. BIM also reveals a negative Yield exposure in the US (weighted towards low yielding stocks in the US) that cannot be identified by a more generic global style framework.

Not surprisingly, the differences in identified portfolio exposures result in significantly different risk forecasts. Table 3 compares the decomposition of risk for this portfolio in Barra’s classical Global Equity Model (GEM) and in the Barra Integrated Model (BIM).

In the GEM analysis, there is no predicted risk from industries or currencies. A small amount of risk comes from style biases. Although the sample portfolio has no active country exposures, a moderate amount of country-related...
The additional insights revealed by the Barra Integrated Model can have a material impact on investment decisions and portfolio results. In addition to this more accurate and action-able decomposition of risk, BIM also provides more accurate estimates of the level of tracking error. Our testing confirms that for a variety of portfolios and time periods, BIM’s predictions are more accurate.\(^2\)

In our example, BIM predicts a tracking error almost 20% greater than that predicted in GEM. As important, BIM explain a much larger portion of tracking error as a result of its local common factors—around 30% of the portfolio’s tracking error can be attributed to purely local sources that are not explicitly captured by the classical global model.

The results and implications of this simplified example hold true for more realistic portfolios. BIM benefits portfolio managers of single market, regional and global portfolios, and is especially enlightening for emerging market portfolios. Emerging equity markets are less integrated with global trends and cycles, and

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1. BIM’s bottom-up construction eliminates the need for generic “country” factors. Within-market sources of risk are more specifically categorized under style, industry and currency.

are predominately characterized by unique local influences. For these developing markets, BIM’s local factors are extremely important in accurately predicting and characterizing risk.

Hedge funds also benefit immensely from BIM’s granularity. Many strategies seek to hedge market risk almost completely, and instead contain large industry or style biases and stock specific bets. A classical global framework assumes all local effects are completely captured by a single market factor, and correlations between similar industries in different countries are all equal to one. This assumption will be inaccurate for all industries, and will be especially misleading for industries that are very dependant on their local environment. For example, construction and real estate industries are particularly sensitive to local economic conditions, fiscal policies and regulations.

While the accuracy of forecasted returns is a very important element of successful investment management, accuracy alone is not sufficient to produce consistent performance over the long-run. A hedge based on the faulty assumption that Hong Kong real estate is equivalent to Swiss real estate can yield hazardous results. By capturing the commonalities and differences in common factors, BIM is able to accurately estimate cross-market relationships.

BIM provides not only a more accurate forecast of risk, but additional insights into true sources of that risk. Through its bottom up construction BIM is able to isolate stock specific risk from common factor risk, and disentangle the impact of every local industry and style decision. This more precise control empowers portfolio managers to manage intended and eliminate unintended sources of risk, therefore improving the consistency of portfolio returns.

Unified and Consistent Framework for Analysis

Barra’s individual country models, such as the US Equity Model or the Japanese Equity Model, offer a high degree of precision and accuracy for their respective markets, but cannot analyze risk for multi-national portfolios. The bottom up construction and unified framework of the Barra Integrated Model is extremely valuable to portfolio managers, risk officers and chief investment officers seeking to analyze risk across different countries, regions and asset classes. Many regional portfolio managers are tasked with relatively narrow mandates that require cross-country risk analyses, such as a Latin American Equity portfolio or an Asia ex-Japan Equity portfolio. Often dominated by local and regional rather than global influences, the risk of these portfolios cannot be accurately and completely characterized by a broad global model. Alternatively, the regional portfolio could be analyzed in multiple country pieces with the relevant single market model—but this cumbersome, piecemeal approach does not take into account the cross-market relationships that often have significant diversifying or concentrating affects on the portfolio. BIM is the ideal solution for these regional portfolio managers, delivering both the cross-market coverage and local market depth needed for a complete understanding of a regional portfolio.

Chief investment officers and risk managers also benefit from the breadth and depth of BIM’s unified framework. In many global asset management organizations, domestic portfolio managers rely on detailed local models, while broad global models are used for multi-national portfolios. Risk oversight and enterprise risk...
analysis may be conducted with a third, more
generalized model to span both countries and
asset classes. The analysis of the same portfolio
in each of these three models will yield poten-
tially dramatically different results. This marked
lack of consistency is akin to viewing some
portfolios with a microscope and others with a
telescope.

While this paper has focused on the equity
portion of the Barra Integrated Model, BIM
covers all major asset classes.1 This single
model can be used for portfolio construction,
risk analysis and enterprise risk management
across managers, countries, products and asset
classes, providing a consistent methodology
and language for communication throughout
the organization. The Barra Integrated Model
provides a new, significantly improved level for
each part of the risk and portfolio management
process.

Exploiting Local Factors in Return Forecasting

The detailed local risk and return information
delivered by the Barra Integrated Model can
also be exploited in the research of active
strategies. In his paper “The Fundamental Law
of Active Management”4, Richard C. Grinold
demonstrates that a manager’s active Sharpe
Ratio (called Information Ratio) can be explained
by two components: strategy breadth and fore-
casting skill. The breadth of a strategy is the
number of independent forecasts made by the
manager. Skill is described as the accuracy of
those forecasts. A good measure of skill is the
correlation between the ex-ante forecast return
and the ex-post realized return. This statistic is
referred to as the Information Coefficient.

While the accuracy of forecasted returns is a
very important element of successful investment
management, accuracy alone is not sufficient to
produce consistent performance over the long-
run. For example, a manager who can accurately
forecast the return of only a single security in
each period must have an unrealistic degree of
certainty about this one forecast. Expanding
the breadth of the strategy by increasing the
number of forecasts allows the manager to
lower the required level of certainty, reduce risk
through diversification and produce more con-
sistent results.

Grinold (1989) also observes that if the correla-
tion between two sources of information is less
than one, then combining these sources will
lead to information synergies: the combined
information coefficient will be greater than the
information coefficient of each source alone. In
other words, diversification adds value in alpha
construction as well as in portfolio construction.

Where can we look for new sources of return?
Managers can focus on individual securities or

<table>
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<th></th>
<th>US</th>
<th>UK</th>
<th>Combined 70/30</th>
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<tr>
<td>Active Return (% per year)</td>
<td>1.55%</td>
<td>1.20%</td>
<td>1.44%</td>
</tr>
<tr>
<td>Active Risk (% per year)</td>
<td>2.05%</td>
<td>2.10%</td>
<td>1.72%</td>
</tr>
<tr>
<td>Active Sharpe Ratio</td>
<td>0.76</td>
<td>0.57</td>
<td>0.84</td>
</tr>
</tbody>
</table>

1 See Dan Stefek, The Barra Integrated Model,” Barra Research Insights, 2002, for a complete description of the Barra Integrated

on common factors: macro-economic variables, industries, styles and countries. The bottom-up common factor framework of BIM allows investment managers to disentangle global and local sources of return. This identifies not only a greater number of investment opportunities, but uncovers potentially more lucrative ones as local trends often receive less attention than global phenomena.

Table 4 illustrates how combining different local information sources increases the active Sharpe Ratio of a strategy. First, we established a signal to predict the return of BIM’s US and UK local factors. We then constructed asset level forecasts based on security exposures to each local factor. Finally, we constructed three portfolios subject to beta neutrality. The first two portfolios were created using US and UK assets only, respectively, while the third portfolio is a mix of 70% US assets and 30% UK assets.5

With an active Sharpe Ratio of 0.76, the US strategy was clearly more successful than the UK strategy, which had an active Sharpe Ratio of 0.57. However, the combined strategy produced superior results when compared to either of the single country strategies. By merging both return strategies in a single portfolio, we benefit from information synergies and diversification and obtain a superior risk-adjusted return.

Conclusion

The Barra Integrated Model constitutes a major step forward in modeling global equity risk. The union of more than 40 highly detailed single market models, BIM offers an unprecedented level of both breadth and depth in coverage and analysis.

Through its bottom up construction, BIM captures local market behavior in detail and improves the accuracy of risk forecasts. Its granularity in capturing intra- and inter-market relationships provides important and actionable insights to portfolio managers, and allows for more precision in managing opportunities for diversification and hedging. This increased level of control empowers portfolio managers to avoid surprises and produce more consistent results. The breadth and depth of BIM’s coverage and analysis is ideal for regional portfolio managers, previously forced to take a view of risk that was far too broad or far too narrow to be useful for a regional mandate. The cross-market and cross-asset class framework of the Barra Integrated Model is invaluable to risk officers and chief investment officers seeking a unified and consistent view of risk across the organization. Finally, insight in the behavior of BIM’s many local factors open up new dimensions for return forecasting. Identifying and combining new sources of return information can reveal new active strategies and improve risk-adjusted returns. These practical benefits of accuracy, unity and new return sources yield one ultimate result: consistently superior risk adjusted returns.

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5 This example was conceived by Anton Puchkov for Barra’s 27th annual research seminar, and presented in the section on “Sources of Return in Global Investing”, 2003.
Since 2000, there has been an increase in the number of stocks with high predicted betas in the US.\(^1\) Resulting from changes in the risk characteristics of a few industries, this phenomenon has implications for hedging strategies as well as portfolio diversification.

**High Betas Get Higher**

Figure 1 illustrates the evolution of the median beta as well as the 5th percentile and 95th percentile betas from the US Short-Term Model from 1998 to 2003.\(^2\)

While the median and the 5th percentile remained relatively stable over this five-year period, the 95th percentile has increased dramatically since 2000. The 95th percentile beta increased from approximately 1.5 in early 2000 to 2.5 in early 2002, stabilizing around 2 in mid-2002. The 95th percentile for historical betas, included in Figure 1, confirms this upward trend. Because historical betas (especially for stocks with a limited history of returns) are very sensitive to outliers, it is not surprising to observe a higher 95th percentile.\(^3\)

**Figure 1**

Predicted beta from the Short-Term US Equity Model

To determine the sources of this phenomenon, we examined the characteristics of the 5% of stocks with the highest betas in the US estimation universe. Interestingly, Technology stocks dominate this group.\(^4\) Figure 2 shows that the proportion of this group categorized as Technology stocks rose dramatically from early 1999. By September 2001, almost all of the

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\(^1\) This phenomenon was originally brought to our attention by Richard Leite and Wai Lee from Credit Suisse Asset Management.

\(^2\) Includes all securities in the US estimation universe from January 1998 to February 2004. Throughout this commentary we use data from the US Short-Term Model.

\(^3\) A minimum of 24 months of return was required to compute historical beta.

\(^4\) We define the Technology group as stocks for which the USE3S primary industry is Semiconductors, Computer Hardware, Computer Software, Wireless or Internet.
stocks in the top 5% were Technology stocks.

Figure 3 illustrates the mean beta for industries in the Technology sector. Each of these industries follows the same general pattern—a rapid increase through 2000 and 2001, followed by a sharp decline in 2002. This pattern is particularly marked for Internet stocks.

Note that technology is not the only sector for which betas have increased. Recall from Figure 2 that the proportion of Technology stocks in the top 5% group begins to fall off in late 2001, indicating the emergence of another high beta industry group. The average beta for Airline and Hotel stocks surged following September 11th, 2001. Figure 4 shows that Barra’s predicted betas were able to pick up very rapidly the structural changes implied by the September events. Historical betas based on monthly or even weekly returns would be much slower in recognizing such new information.

**The Source of Rising Betas**

A technology bubble and a terrorist attack are two very different situations, but both resulted in rising betas for stocks at the center of the phenomenon. To better understand the catalysts of a rising beta, we can break this metric down into two distinct components. Recall that beta is equal to:

\[
\beta = \rho \frac{\sigma_{\text{asset}}}{\sigma_{\text{index}}}
\]

where \(\rho\) = the correlation of stock return with the market return, and

\[
\frac{\sigma_{\text{asset}}}{\sigma_{\text{index}}} = \text{relative volatility, or the ratio of asset volatility to index volatility.}
\]

An increase in the beta of a stock (or industry) can be attributed to a rise in correlation or a rise in relative volatility.

Figure 5 shows these components of beta for the Airline industry. The left axis plots the average correlation with the market index, while the right plots the relative volatility of the industry.
Except for the period around September 2001, correlation and relative volatility tend to move in opposite directions. By definition, the less a stock’s return is explained by market movements, the lower the correlation. Assuming no change in the distribution of market returns, an increase in relative volatility that originates from an increase in residual risk (that is not explained by the market) will be accompanied by a reduction in correlation with the market index.

September 11th changed this dynamic. Airlines experienced a sharp and simultaneous increase in both correlation and relative volatility. The conjunction of those two elements fueled the dramatic beta increase depicted in Figure 4.

Over the same period, Internet stocks had an entirely different experience. Figure 6 illustrates the steady decline in the relative volatility of Internet stocks since 2000—mean relative volatility plunged from seven times that of the S&P500 in 2000 to only four times the volatility of the index in 2003. All else being equal, this reduction in relative volatility would have resulted in lower betas for Internet stocks.

However, the decline in volatility was accompanied by an even more pronounced increase in the industry's correlation with the S&P500. The mean correlation of Internet stocks relative to the S&P500 more than doubled over the one-year period from April 2000 to April 2001. In 2002, the industry’s average correlation with the market index remained stable, while the relative volatility of Internet stocks continued to decline. As illustrated in Figure 7, the average beta of Internet stocks subsequently fell back below 2.
Implications for Portfolio Management

An increase in correlation with the market index exemplified above implies that market returns have become more important in explaining stock returns for some industries. Figure 8 and Figure 9 illustrate the increasing proportion of predicted variance attributable to the S&P500 for Technology and Airline stocks, respectively.

Because a larger portion of risk is market related, beta has become a powerful tool for hedging the risk of investments in these industries. In December of 2000, an average of 12% of a Technology stock’s predicted variance could be removed with an appropriate beta hedging policy. By December 2003, this proportion had increased to 23%. This phenomenon was even stronger for Airline stocks, where the proportion of market variance increased from 9% in December 2000 to 30% in December 2003.

The rising importance of market related risk also has implications for diversification. US market risk cannot be diversified away by investing across many US equity securities, only through investment in other markets. As a result, the amount of risk reduction one can expect from domestic diversification has been considerably reduced for Airline and Technology stocks.

Conclusion

We have witnessed a growing number of high beta stocks since 2000. Due to rising correlation with the market, the betas of Technology stocks increased dramatically during the fall of the dot-com bubble. The average beta for Airline and Hotel stocks soared after September 11th due to a simultaneous increase in both correlation and relative volatility. The growing importance of market related volatility in these industries implies a greater efficacy of beta strategies and a decrease in the benefits of domestic diversification.

Our discussion has focused on the S&P500 as a representative market index. Because the S&P500 has a strong large-cap bias, “market related” risk can be partially diversified by investing in small-cap securities.
We present this partial listing of research papers, articles and books published during the last quarter. To access the complete Barra Research Database online, please log on to www.barra.com/research.

**Multi-Asset Class: Risk Modeling and Measurement**

KEHR, Carl-Heinrich

**Equity: Risk Modeling and Reporting**

CONNER, Greg
“Style Analysis in an Evolving Europe,” IPE, Fall 2003.

CONNER, Greg

SÉNÉCHAL, Edouard

**Equity: Investing**

CASHION, Daniel and HANADA, Rohtas

KEHR, Carl-Heinrich

SOMERVILLE, Sara and SÉNÉCHAL, Edouard
Equity: Performance Measurement and Attribution

GILFEDDER, Neil and ZHELEZNYAK, Alexander

LAKER, Damien

LAKER, Damien

LAKER, Damien

Fixed Income: Risk Modeling

BREGER, Ludovic

GOLDBERG, Lisa and KERCHEVAL, Alec and ANDERSON, Greg and MILLER, Guy

GIESEKE, Kay

GIESEKE, Kay

GOLDBERG, Lisa

Fixed Income: Credit Risk

BACKSHALL, Tim

BACKSHALL, Tim

BREGER, Ludovic and GOLDBERG, Lisa and CHEYETTE, Oren

BREGER, Ludovic and GOLDBERG, Lisa and CHEYETTE, Oren

CHEYETTE, Oren and TOMAICH, Tim

GIESEKE, Kay

GIESEKE, Kay and GOLDBERG, Lisa

GIESEKE, Kay and GOLDBERG, Lisa
KERCHEVAL, Alex, GOLDBERG, Lisa and BREGER, Ludovic

PODURI, Vijay

The complete Barra Research Database is available at http://www.barra.com/research
AWZ, Inc. wants to buy out Krill, Ltd. for strategic reasons. Before announcing the deal the JAWZ lawyers and company analyst have deemed it necessary that JAWZ hold a net long position of 100,000 shares of Krill to negotiate a fair price and repel any possible counter-offers.

The JAWZ accounting team has notified the executive committee that the company has exactly enough cash to pay for the 100,000 shares now selling at $10. However, the trading team informs the committee that making the purchase in one day is impossible since Krill shares are rather illiquid. To make things worse, the business strategy group reveals a more considerable problem. They discover that BIG, Inc., a powerful and extremely rich corporation, is intent that Krill remains as an independent company going forward. For this reason, BIG is monitoring the trading volume and price changes of Krill stock for any suspicious activity that might alert them to a possible buyout. If the analysts at BIG get a whiff of the deal before JAWZ has accumulated the necessary 100,000 shares then BIG will put the deal in jeopardy.

The JAWZ executive committee is desperate since this deal is strategically crucial to the company. The committee calls upon a former BIG analyst who now works for JAWZ to provide tactical information about BIG’s monitoring practices. However, the lawyers suggest that the employee give advice only, to avoid any possible legal technicalities. The ex-BIG employee gives the following advice:

“Make only one transaction of 4,000 shares per day and do not let the closing price change from the close on the day before your first transaction.”

The executive team becomes confused and discouraged, since this advice seems vague and unpractical. The trading team is told to work with the quantitative group to devise a strategy using the advice given. In a subsequent meeting, the trading team informs the quantitative group that they can manipulate the trading mechanism to ensure that JAWZ makes the last transaction on Krill stock before the close of trading. They also specify that short selling is possible as long as the total dollar amount of the short position is kept in a separate cash account.

Suppose Krill closed at $10 yesterday and the JAWZ executive committee wants to start a strategy today. What strategy can the quantitative group implement that has a chance at buying the 100,000 Krill shares without alerting BIG?

What is the least amount that JAWZ should borrow (to put in a margin account) in order to
have a 75% chance of buying the 100,000 shares of Krill?

Suppose a merger arbitrage hedge fund is suspicious of the takeover. The hedge fund has decided to buy Krill stock on any day that JAWZ has a net long position of Krill shares. Due to illiquidity this increase in demand for Krill stock will increase the probability of the stock being higher near close by 1/20. What is the new probability of success of the strategy given the amount JAWZ had borrowed?

Now suppose the amount of Krill stock bought by the hedge fund depends on the amount of shares owned by JAWZ and the increase in higher price probability near close becomes \(x/100\), where \(x = \text{(#shares owned /transaction amount)}\). What is the new probability of strategy success in this case?

Brainteaser from Last Issue

Amoebas reproduce asexually by splitting into identical copies of themselves. Each generation represents one time period. In one time period, every amoeba independently has a 50% chance of splitting into two and a 50% chance of dying.

Let generation 0 consist of 1 amoeba. What is the chance of having amoebas alive at generation 2?

What is the probability \(p(t)\) of having at least 1 amoeba alive at time \(t\)? It would suffice to give a recursive formula for \(p(t)\) in terms of \(p(t-1)\).

What is the probability that the amoebas’ progeny exist forever?

After some genetic “improvement”, a new species of amoeba is more successful at reproducing, and now has an 80% chance of splitting into two and a 20% chance of dying in one time period.

- What is the probability of having 1 amoeba alive at time \(t\) in this case? Again, a recursive formula would suffice.
- What is the probability that these amoebas’ descendents exist forever?

If instead there are equal probabilities (33%) that an amoeba will split, die, or stay the same, then what is the probability that these amoeba’s descendents exist forever?

In nature there is a tendency to maximize the “fitness” of genes. Taken to its logical conclusion, this could lead to some interesting adaptations not beneficial to the “individual”.

For example, all other things being equal, a sacrificial gene that caused me to die in order to save at least two of my siblings from certain death should theoretically thrive in a population, since each of my siblings have a 50% chance of sharing that gene, so on average my death has saved at least another copy of that gene for future generations.

Based on this reasoning, and all other factors remaining equal:

- How many first cousins should I sacrifice myself for?
- How many second cousins?
- How many half siblings?
- How many step siblings?
- How many identical twins?

Some animals which are very strongly related, such as soldier ants, do display true sacrificial behavior on behalf of their colony. Ant nests tend to have one mature female queen, who made one mating flight when young and stored the sperms from one male for the rest of her life. Some of her eggs are fertilized, and become female ants, with potential to develop into soldiers, workers etc, or even new queens. They have two sets of chromosomes. The rest
are left unfertilized and develop into male ants with only a single set of chromosomes, and no "father". This leads to each sperm of the male being identical, containing this one set of chromosomes.

Based on this, and if we define "relatedness" as the chances that a gene from organism A exists in organism B, then the male is 100% related to his mother, but the mother (queen) is only 50% related to her son.

- How related is a daughter to her mother?
- How related is a mother to her daughter?
- How related are two sisters (each daughters of the queen)?

Solution

The probability of having no amoebae at the second time period can be calculated by considering the possible outcomes and their probabilities.

The chance of having no amoebae after the first time period is 1/2.

Otherwise, you will have 2 with probability 1/2.

The conditional probability of these both dying after the second period is 1/2 · 1/2.

Hence total probability of none alive is (1/2) + (1/2 · (1/2 · 1/2)) = 5/8, so the probability of having at least one alive is 1 - 5/8 = 3/8.

It is very easy to get lost in a mass of formulae to calculate the probability of having at least 1 amoeba alive at time t. There are 2 conceptual leaps that can be made to vastly simplify the problem.

Firstly, note that p(t) = 1 - prob (all amoebas are dead at time t) which we shall define q(t). Then consider the amoebas’ future offspring in terms of the next generation.

The probability of no offspring of the generation 0 single amoeba being alive in t time periods is equal to the probability of its direct first generation offspring having none of their offspring alive in t-1 time periods.

Because the amoebas are independent, the probability qN(t) of the offspring N amoebas (N is any integer) of a single generation having no offspring left in t time period is q(t)^N.

Since a single amoeba has either 2 or 0 offspring in the next generation, we only need to consider these instances.

Hence q(t) = [Prob (no amoebas in next generation) · (q(t-1))^0] + [Prob (2 amoebas in next generation) · (q(t-1))^2]. Note any number raised to power 0 is 1.

Hence q(t) = [1/2 · 1] + [1/2 · q(t-1)^2]

Substituting for p(t) = 1 - q(t),

1 - p(t) = 1/2 + [1/2 · (1-p(t-1))^2]

gives p(t) = p(t-1) - 1/2 · (p(t-1)^2)

The probability of the offspring lasting forever is 1 - prob (offspring do not last forever).

Furthermore, the probability of the offspring of the generation 0's amoebas offspring becoming extinct is equivalent to its first generation's offspring all fathering extinct lines. Hence, in the formula q(t) = 1/2 + (1/2 · q(t-1)^2), in this case we can set q(t) = q(t+1), which we shall define as q. Then we have a quadratic equation q = 1/2 + 1/2 · q^2 which factorises as (q - 1)(q - 0) = 0 hence q = 1... (i.e., there is a 100% probability that the offspring will become extinct at some point in the future, or 0 chance that the progeny will last forever.

In the case of a 0.8 chance of a split, and 0.2 chance of dying, we can similarly define q(t) as above, with

q(t) = [0.2 · 1] + [0.8 · q(t-1)^2]
so
$q(t) = 0.2 + 0.8q(t-1)^2$
$p(t) = 1.6p(t-1) - 0.8p(t-1)^2$

Again, to see the chance they exit forever, set
$q(t) = q(t-1) = q$
so
$4q^2 - 5q + 1 = 0$
which solves as $q = 1/4$, hence probability that
progeny exist forever is 3/4.

If there are equal probabilities of 1/3 that an
amoeba will split, die or stay the same, then
we have
$q(t) = 1/3 + (1/3)q(t-1) + (1/3)q(t-1)^2$.
setting $q(t) = q(t-1) = q$ as before, and solving,
we again get $q = 1$, so 0 as the probability that
the descendants exist forever in this case.

For the next section, the reasoning means that
we need to take the inverse of the probability
that the sacrificial gene is in the relation to give
the number of such relations we should sacrifice
ourselves for. “Relatedness” can be calculated
by considering that one shares 1/2 their genes
with a parent, and half with a sibling.

We share on average 1/8 of such genes with
our first cousins, so the answer is 8.

The probability of a given gene being in a
second cousin is 1/16, so the answer is 16.
It is 1/4 for half-siblings, giving answer 4.

We are not genetically related to step-siblings,
so we should not sacrifice ourselves for any
number.

Finally, since we share all our genes with
identical twins, we should sacrifice ourselves
for just 1.

In the final section, the daughter is 50% relat-
ed to her mother, since the daughter has 50%
of her genes from her mother, and 50% from
her father, so a randomly chosen gene is only
50% likely from her mother.

The mother is 50% related to her daughter,
since a gene from the mother has a 50% chance
of being on the egg that created the daughter.

Two daughters of the queen are 75% related. To see this, consider a randomly cho-
osen gene from the first sister. This has a 50%
chance of being from the mother, and a further
50% chance that the mother gave it to the sec-
ond sister, so there is a 25% chance that the
gene is shared via the mother.

Also, there is a 50% chance that the gene came
from the father’s sperm. Since all his sperm are
genetically the same, the second sister is cer-
tain to share the gene in this case, giving a 50%
chance the gene is shared via the father.

So we have 25% + 50% = 75%.
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