The European credit market, consisting mainly of euro and sterling denominated debt, is second only to the US domestic market in terms of size, influence and liquidity. Not surprisingly, European securities are becoming common in global portfolios. The recent turmoil in credit markets has shown once again that understanding risk is or should be a critical aspect of portfolio management. However, as the European credit market is a mosaic of widely different instruments, issuers, and currencies, identifying and forecasting the risk of European fixed income securities is not a simple task.

This chapter will take the reader through the process of building a European risk model and discuss the important sources of risk in generic fixed income portfolios. Our intention is not to cover the whole spectrum of securities but to address some typical modeling challenges such as accommodating different benchmarks and securities, and providing a wide coverage without compromising accuracy. With a general framework in place, the model can be easily extended to cover more markets or bond types.

The author thanks Jean-Martin Aussant, Oren Cheyette, and Darren Stovel for insightful comments and suggestions on how to improve this chapter.
A FRAMEWORK FOR UNDERSTANDING AND MODELING RISK

This discussion covers the main factors affecting bond returns in the European fixed income market, namely, the random fluctuations of interest rates and bond yield spreads, the risk of an obligor defaulting on its debt, or issuer-specific risk, and currency risk. There are also other, more subtle sources of risk. Some bonds such as mortgage-backed and asset-backed securities are exposed to prepayment risk but such instruments still represent a small fraction of the total outstanding European debt. Bonds with embedded options are exposed to volatility risk. However, it is not apparent that this risk is significant outside derivatives markets.

A detailed understanding of correlations between asset returns is required to accurately estimate the risk of a portfolio. Unfortunately, estimating correlations directly is in practice impossible as unknowns severely outnumber observations even in relatively small portfolios. The standard solution is to decompose the portfolio’s vector of asset returns using market-wide common factors:\(^1\)

\[
\mathbf{r}_{\text{excess}} = \mathbf{X} : \mathbf{f} + \mathbf{r}_{\text{specific}} \tag{X.1}
\]

where
\[
\begin{align*}
\mathbf{X} &= \text{the matrix of asset exposures} \\
\mathbf{f} &= \text{the vector of factor returns} \\
\mathbf{r}_{\text{specific}} &= \text{the vector of asset residual returns not explained by factors, or specific returns idiosyncratic to individual assets}
\end{align*}
\]

Decomposing returns is a key step in identifying, understanding, and modeling the sources of risk that are at work in the market. It is also crucial in understanding risk exposures.

We begin our analysis by writing the excess returns of assets in a portfolio as:

\[
\mathbf{r}_{\text{excess}} = \left( \mathbf{r}_{\text{IR}} + \mathbf{r}_{\text{curr}} + \mathbf{r}_{\text{spread factor}} + \mathbf{r}_{\text{specific}} \right) \tag{X.2}
\]

where
\[
\begin{align*}
\mathbf{r}_{\text{IR}} &= \text{the vector of returns due to changes in interest rates}
\end{align*}
\]

Note that the decomposition implicitly ignores the predictable component of return that is irrelevant for risk modeling purposes. The return common horizon will be one month in most cases. Although daily or even weekly returns would provide a much larger data set, they are also on average much more sensitive to noise in bond data. We will also see in what follows that it is sometimes possible to use returns over a shorter time horizon.

If the return factor model adequately accounts for common factors, then the specific returns are uncorrelated and we can write portfolio risk as:

$$\sigma^2 = \Sigma_{b}$$  \hspace{1cm} (X.3)

with

$$\Sigma = T_X \cdot \Phi \cdot X + \Delta$$  \hspace{1cm} (X.4)

where

- $h$ = the vector of portfolio holdings
- $\Sigma$ = the covariance matrix of asset returns
- $\Phi$ = the covariance matrix of factor returns
- $\Delta$ = the diagonal matrix of specific variances

Equation (X.4) will yield active risk forecasts when $h$ is a vector of active holdings.

The data that can go into computing factor returns will of course depend on what the factors are. It can include bond and index level data as well as currency exchange rates. Assume that we have the factor return series. To construct covariances, we could postulate that the underlying random processes are time stationary and compute covariances using equally weighted factor returns. We actually know that mar-

---

2 Some market idiosyncrasies such as settlement conventions are an important part of a valuation model but irrelevant to a risk model.
3 For instance, short-horizons spread returns observed for high-grade corporate bonds are small and are typically very noisy.
kets change over time and that recent data are more representative of current market conditions than are older data. A simple method for accommodating this fact consists in exponentially weighting factor returns to calculate the covariance matrix. The relative weight of returns from time $\tau$ in the past relative to the most recent returns is $e^{-t/\tau}$, where $\tau$ is a time-decay constant. The optimal time constant $\tau$ can be obtained empirically using, for instance, a maximum-likelihood estimator. However, series that are particularly volatile may require a different treatment (see for instance Currency Returns section).

Much of the art of constructing a model goes in choosing relevant factors. Note that factors are descriptive and not explanatory. In other words, they allow for forecasting risk without necessarily being linked to the forces that really drive interest rates or returns. Let’s now proceed with a discussion of several classes of factors.

**INTEREST RATE RISK**

Interest rate or term structure risk stems from movements in the benchmark interest rate curve. Excluding exchange rate risk, it is the main source of risk for most investment-grade bonds. Any reasonable model will include markets that are stable and actively traded. A typical coverage, taken from JP Morgan GBI Broad Index, is shown in Exhibit XX.1. Note the presence of two emerging markets.

Building a term structure risk model for the European market involves choosing several benchmarks—at least one for each currency. A recent complication is that domestic government yields are no longer the universal choice. The LIBOR/swap curve has recently emerged as the euro zone preferred benchmark due to the absence of a natural sovereign yield curve and the growing liquidity and transparency of swap

**EXHIBIT X.1** European Markets in JP Morgan GBI Broad Index as of January 1, 2003

<table>
<thead>
<tr>
<th>Country</th>
<th>Country</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>Greece</td>
<td>Portugal</td>
</tr>
<tr>
<td>Belgium</td>
<td>Hungary</td>
<td>Spain</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>Ireland</td>
<td>Sweden</td>
</tr>
<tr>
<td>Denmark</td>
<td>Italy</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Finland</td>
<td>Netherlands</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>France</td>
<td>Norway</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Poland</td>
<td></td>
</tr>
</tbody>
</table>

$^4$ The half-life is $\tau \ln 2$. 
curves. However, many markets continue to trade primarily with respect to the government benchmark. In some emerging markets, the absence of a liquid market for sovereign debt makes the LIBOR/swap curve the only available benchmark. A simple approach used at Barra and which permits alternative views is to use the sovereign term structure as local benchmark whenever possible and include a swap spread “intermediate” factor that can be added to the sovereign-based interest rate factors to allow interest rate to be expressed with respect with the swap curve. This swap spread factor will be described in more detail in the next section. In markets where the benchmark is already the LIBOR/swap curve, there is obviously no need for a swap factor.

The existence of a Euro zone born from the union of several legacy markets introduces an additional modeling challenge. More than one domestic government is issuing euro-denominated debt, and although yields have converged, some differences clearly remain that suggest building a set of factors for each legacy market. (See Exhibit XX.2 for some examples of sovereign term structures within the Euro zone.) Some bonds also need to be analyzed almost completely independently of other assets. This is the case for Inflation Protected Bonds (IPBs) denominated in euro or sterling, which offer investors a “real” inflation-adjusted yield. Such securities are weakly correlated with other asset classes and are exposed to a set of IPB-specific interest risk factors simi-

EXHIBIT X.2   Examples of Sovereign Term Structures within the Euro Zone on July 31, 2002
lar in nature to the conventional interest rate factors but derived from IPB data and real yields.

What should the interest rate factors be? Key rate durations, which are rate changes at the term structure vertices, seem a natural and somewhat appealing choice. However, because rates for different maturities are highly correlated, using so many factors is unnecessary, and causes difficulty with spurious correlations. Anywhere from 90% to 98% of term structure risk can in fact be modeled using only three principal components commonly referred to as *shift*, *twist*, and *butterfly*. The principal component analysis is now a fairly standard approach that we describe in more details in the Appendix. Exhibit XX.3 shows examples of factor shapes. Note how principal components derived from Portuguese sovereign euro-denominated debt are very different from the German shapes. Such large differences within the euro zone confirm the need for a different set of factors in each legacy market.

Shift, twist, and butterfly volatilities are shown in Exhibit XX.4. Quasi-parallel shifts in the term structure are the dominant source of risk in all cases with volatilities ranging from 35 to 200 basis points per year. In spite of these large differences, term structure risk is relatively homogeneous across most markets and in particular within the euro zone. Note,
again, that the differences in factor volatilities are sufficiently large to justify building separate legacy factors. As expected, the largest volatilities are observed for emerging market benchmarks, Czech Republic being the riskiest market. And not surprisingly, real yields appear to be more stable than their non-inflation protected sovereign counterparts.

EXHIBIT X.4  Interest Rate Factor Volatilities on July 31, 2002

An alternative but less accurate approach would be to build a unique set of EMU interest rate factors and capture each legacy market idiosyncrasies with a spread factor.
EXHIBIT X.5  Examples of Interest Rate Risk Breakdown

<table>
<thead>
<tr>
<th>Exposure Risk (bp/yr)</th>
<th>Shift</th>
<th>Twist</th>
<th>Butterfly</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Federal Republic of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany 8.5% 07/16/07</td>
<td>4.9</td>
<td>-0.3</td>
<td>-2.9</td>
<td>310</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Federal Republic of</td>
<td>10.8</td>
<td>18.7</td>
<td>10.6</td>
<td>680</td>
</tr>
<tr>
<td>Germany 4.75% 07/04/28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.2</td>
<td>15.0</td>
<td>10.3</td>
<td>1900</td>
</tr>
<tr>
<td>Tchek Republic 6.95%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01/26/16</td>
<td></td>
<td></td>
<td></td>
<td>2700</td>
</tr>
</tbody>
</table>

The interest rate risk of any given bond will depend on the bond’s exposures to the factors and on correlations between factors. Exhibit X.5 gives detailed risk decompositions for three sovereign bonds. The typical annualized risk of a straight bond issued by the Federal Republic of Germany varies from about 200 to 300 bp to over 800 bp, depending on its duration. At the other end of the spectrum, the interest rate risk of a bond issued by the Czech Republic can reach as much as 3000 bp, which exceeds the risk of most speculative corporate issues in developed markets. Clearly, such extreme cases will require special attention when controlling risk.

SPREAD RISK

Until fairly recently, outside the US, UK, and Japan, there were relatively few tradable non-government bonds. The recent explosion of the global corporate credit market now provides asset managers with new opportunities for higher returns and diversification. Unlike domestic government debt, however, corporate debt is exposed to spread risk, which arises from unexpected yield spread changes. For modeling purposes, such changes can again be decomposed into a systematic component that describes, for instance, a market-wide jump in the spread of A-rated utility debt and can be captured by common spread factors, and an issuer or bond-specific component. This section discusses model market-wide spread risk, while the next section will address issuer specific spread risk and default risk.

Data considerations are crucial in choosing factors. The choice of factors will be somewhat limited in markets with little corporate debt. Spread factors should increase the investor’s insight and be easy to interpret. Meaningful factors will in practice be somewhat connected to the portfolio assets and construction process and allow a detailed analysis of market risk without threatening parsimony.

Note here how twist-like movements of the Tchek benchmark account for more risk than the shift distortions themselves. A simpler, duration-based model would severely under-forecast risk.
Swap Spread Factors

First, as mentioned earlier, there is usually no universal benchmark in a given market. Again, a possible approach, used in Barra’s models, is to introduce a swap spread factor that describes the average spread between sovereign and swap rates and can conveniently allow spread risk to be expressed with respect to the LIBOR/Swap curve when interest rate risk factors are originally based on the sovereign yield curve.

This same factor can also be used to compute spread risk in markets where there is not enough data to build a detailed credit block. It can also be used in markets where more detailed credit factors are available, but when there is not enough information to expose a bond to the appropriate credit factor. As we will see in what follows, this will be the case when a euro or sterling-denominated corporate bond is not rated. Based on the observation that bonds with larger spreads are on average more risky, Barra’s model assumes the following exposure to the swap factor:

\[ x = D_{\text{eff}} + (\alpha - 1) \cdot D_{\text{spread}} \]

with \( \alpha = \max\left[1, \left(\frac{\text{OAS}}{S}\right)^\gamma\right] \)  

where

- \( D_{\text{eff}} \) = the bond effective duration
- \( D_{\text{spread}} \) = the bond spread duration
- \( \text{OAS} \) = the bond spread
- \( S \) = the swap spread
- \( \gamma \) = a scaling exponent determined empirically and equal to 0.6

At the time of this writing, corporate bonds denominated in currencies others than euro and sterling are only exposed to the local interest factors and if it exists, the swap factor. This swap factor is roughly equivalent to a Financial AA spread factor, as the bulk of organizations that engage in swaps are AA-rated financial institutions. The swap model is coarser than the two local credit models discussed in the next section, but it performs adequately because spread changes are highly correlated within markets.

Swap spread volatilities for several currencies are shown in Exhibit XX.6, with values that vary from about 15 bp/yr to 40 bp/yr. Also shown are the resulting spread risks in the euro and sterling markets for several rating categories. We will see further below that the swap model predicts reasonably accurately both the absolute magnitude of the spread risk in each market and their relative values.
EXHIBIT X.6  Swap Spreads
Panel a: Swap Spread Annualized Volatility for Markets Covered in the Model

Credit Spread Factors
The euro and sterling markets are broad and liquid markets. Accurately modeling spread risk in these two markets requires market-dependent, “credit blocks.”

Various considerations drive the choice of spread factors. Factors built on little data can end up capturing a large amount of idiosyncratic risk and be representative of a few issuers rather than the market. A cor-
ollary is that it is often wiser to avoid building separate factors for thin industries. Spread factors should be meaningful for the investor, and somehow be related to the process of constructing a portfolio.

An obvious and natural approach is to capture fluctuations in the average spread of bonds with the same sector and rating. As the size of the high-yield European bond market is still modest, there is unfortunately not enough data to construct sector-by-rating factors for speculative ratings. The simplest alternative is then to construct rating-based factors. A typical sector and rating breakdown for the euro market is given in Exhibit XX.7.

Note that using market-adjusted ratings as opposed to conventional agency ratings can increase the explanatory power of sector-by-rating spread factors. The idea is to adjust the rating of bonds with a spread that is not too different from the average spread observed within their rating category. For instance, a AA-rated euro-denominated bond with a spread equal to 200 bp would be reclassified into a BBB-rated bond.7

Credit spreads are computed with respect to the local swap curve to accommodate for the swap spread factor.

**EXHIBIT X.7 Example of Sector and Rating Breakdown in the Euro Market**

<table>
<thead>
<tr>
<th>Euros</th>
<th>Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agency</td>
<td>AAA</td>
</tr>
<tr>
<td>Financial</td>
<td>AA</td>
</tr>
<tr>
<td>Foreign sovereign</td>
<td>A</td>
</tr>
<tr>
<td>Energy</td>
<td>BBB</td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
</tr>
<tr>
<td>Pfandbrief</td>
<td></td>
</tr>
<tr>
<td>Supranational</td>
<td></td>
</tr>
<tr>
<td>Telecom</td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>BB</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>CCC</td>
</tr>
</tbody>
</table>

*A non-domestic sovereign bond is exposed to the factor corresponding to its sector and rating. Due to the limited number of high-yield bonds outstanding, non-investment grade factors are only broken down by ratings.

Note that arbitrage considerations imply that the spread risk of issues from the same obligor should be independent of the market. Why then do we need two sets of credit factors? After all, a model with only one set would be more parsimonious. Empirical evidence simply shows that spread risk is indeed currency-dependent, at least for higher credit quality issuers.⁸

Volatilities for selected factors are displayed in Exhibit XX.8. Spread risk in the euro and sterling markets is, on average, comparable. Looking now in more details, sterling factors tend to be more volatile.

than euro factors for AAA, AA, and A ratings, and less volatile for lower ratings. This is a trend already seen in Exhibit XX.6 that confirms that the swap factor would be a simpler but meaningful alternative. Significant differences exist for individual factors that illustrate the need for currency-dependent factors (see for instance the Telecom A and BBB factors). Also note how the high volatilities of the Energy, Utility, and especially Telecom factors reflect the recent problems in these industries.

Each corporate bond will only be exposed to one of these factors, with an exposure that will typically increase with the bond’s maturity. A rule of thumb is that it will be comparable to the bond’s exposure to the shift factor. The spread risk of almost all AAA, AA, and A rated bonds will be less than their interest rate risk, and it is only for BBB rated bonds and in some very specific market sectors such as Energy and Telecoms that spread risk starts exceeding benchmark risk. Spread risk is by far the dominant source of systematic risk for high-yield instruments.

**Emerging Markets Spread Factors**

Emerging debt can be issued either in the local currency or in any other external currencies (i.e., Mexico issuing in euro or sterling). These two types of debt do not carry the same risk, and need to be modeled independently. “Internal” risk was discussed in the interest risk section and we will now address external risk.

A rather natural approach is to expose emerging market bonds to a spread factor. The sovereign spread factor turns out to be a poor candidate as the risk of emerging market debt strongly depends on the country of issue. Exhibit XX.9 shows average Argentinean monthly spread returns from June 30, 1999 to June 30, 2002 for US dollar-denominated debt. The collapse of the peso, the illiquidity of the financial system, and the brutal decline in the economic activity are all reflected in Argentinean returns. Chilean spreads remained virtually unaffected despite a strong economic link between the two countries. As a result, any accurate model will need at least one factor per country of issue.

The amount of data available for building emerging market spread factors is unfortunately scarce. First, there are often at best only a few bonds issued by sovereign issuers in emerging markets. The second problem is that there are mostly US dollar-denominated. Even when some bonds denominated in, say, euro are available, there is generally little returns history. In some cases, we will seek risk forecasts for an

---

9 External debt is more risky than internal debt. If needed, sovereign governments can generally raise taxes or print money to service their internal debt. A shortage of external currencies can be more dramatic. This can be seen in the credit ratings delivered by agencies.
issuer with no history of issuance in a specific currency. Since the risk of an emerging market bond is directly related to the creditworthiness of the sovereign issuer, which is independent of the currency of denomination, we can actually borrow from the history of US dollar-denominated emerging market returns to forecast spread volatilities in other currencies. Spread return data can be obtained from an index such as JP Morgan Emerging Markets Bond Index Global (EMBIG).

Strictly speaking, these factors are applicable only to sovereign and sovereign agency issuers, based on the inclusion criteria for, say, EMBIG if we happen to use this particular index to estimate emerging markets spread factors. However, many issuers of external debt domiciled in these markets carry a risk that is comparable to the corresponding sovereign issuers, so that it is reasonable to use the sovereign factor as a proxy for corporate issuers.

Emerging market spread volatilities are shown in Exhibit XX.10. The spread risk of Latin American obligors tend to be above average, currently largest for Argentinian and Brazilian markets that have a spread risk comparable to a B to CCC-rated euro corporate. The risk of Asian issuers is on the other hand below average and comparable to the interest rate observed in developed markets. We clearly observe a wide spectrum of risk characteristics that confirms the need to build a separate factor for each market.
Specific risk returns are residual returns not explained by common factors. Common factors returns are typically larger than specific returns for higher quality investment-grade instruments; this is no longer the case in the lower portion of the investment grade segment and for high-yield instruments.

One option is to use a CreditMetrics-like model based on transition probabilities reported by rating agencies. The model assumes that specific return variance of any bond can be written as:

$$\sigma_{\text{spec}}^2 = \sum_i p_{i \rightarrow j} [D(s_j - s_i) - r_m]^2 + p_{i \rightarrow d}(1 - R - r_m)^2$$  \hspace{1cm} (X.6)
Transition probabilities are a crucial ingredient to this formula, and more generally, to any credit portfolio model based on ratings. Although agencies such as Standard and Poor’s do report European-specific rates, they are based on a small number of credit events, particularly for low quality ratings, yielding poorly constrained values. Global transition rates are statistically more robust because they are derived from a dataset that covers far more obligors and a longer time period.

The model uses average spread levels observed within each rating category. Since these levels are market-dependent, so is specific risk. Another consequence is that this approach can only be implemented in highly liquid markets, where there are enough bonds to robustly estimate average spread levels—in practice, markets for which we can construct sector-by-rating credit factors.

In markets where there is not enough data to construct a detailed model, a simple solution is to write the specific risk forecast as:

\[ \sigma_{\text{spec}} = (a + b \cdot s)D \]  

where

\( s \) = the bond's spread  
\( D \) = the bond's duration

The two constants \( a \) and \( b \) are fitted in each market using observed residual returns. Typical values for Swiss francs-denominated bonds are on the order of 5 \times 10^{-4} \text{ bp/yr} and 5 \times 10^{-6} \text{ bp/yr}, respectively, if spreads are expressed in basis points.

**CURRENCY RISK**

Currency risk is potentially a large source of risk for global investors that can be handled with a multi-factor model with one factor per currency. Yet, special attention has to be paid in forecasting the variances
and covariances of currency factors due to their high volatility and rapidly changing risk characteristics. The goal is to obtain a model that quickly adjusts to new risk regimes and responds to new data.

Various forms of General Auto-Regressive Conditional Heteroskedastic (GARCH) models have been used to estimate return volatility. Such models express current volatility as a function of previous returns and forecasts. For instance, the GARCH(1,1) model takes the form:

\[ \sigma_t^2 = \omega + \beta(\sigma_{t-1}^2 - \omega) + \gamma(r_{t-1}^2 - \omega) \]  

(X.8)

where

- \( \sigma_t^2 \) = the conditional variance forecast at time \( t \)
- \( \omega \) = the unconditional variance forecast
- \( \beta \) = the persistence
- \( \gamma \) = the sensitivity to new events
- \( r_{t-1} \) = the observed return from \( t-1 \) to \( t \)

The constants \( \beta \) and \( \gamma \) must be positive to insure a positive variance, even if large events occur. For the same reason, the condition \( \beta + \gamma < 1 \) must hold. The higher the sensitivity, the more responsive the model is. The weight given to past forecasts increase with the persistence constant.

Using daily exchange rates as opposed to weekly or monthly exchange rates insures the convergence of GARCH parameters and minimizes standard errors.\(^{10}\) The aggregation formula for monthly GARCH forecasts is:

\[ \sigma_{t,n}^2 = n\omega + \frac{1 - (\beta + \gamma)^n(\sigma_t^2 - \omega^2)}{1 - (\beta + \gamma)} \]  

(X.9)

where \( n \) is the number of business days in a month, typically 20 or 21.

**Exhibit XX.11** shows US dollar versus euro returns from 1994 to 2000. Note how volatility forecasts (gray lines) quickly adjust to periods of small or large returns. The overall currency risk is large compared to interest rate risk. Consider a German government bond with a duration equal to 5 years. A US investor holding this asset is facing an additional currency risk of at least 8% per year in addition to an interest risk of roughly 70 bp \( \times 5 = 3.5\% \).\(^{11}\) The volatilities of several European currencies are plotted in **Exhibit XX.12**, and typically range from roughly 6.5% to 10% per year.

\(^{10}\) This is because daily exchange rate returns represent a much larger dataset than weekly and monthly returns.

\(^{11}\) 70 bp is the German shift volatility reported in **Exhibit XX.4**.
EXHIBIT X.11  US Dollar Against Euro Currency Returns and Volatility

EXHIBIT X.12  Examples of European Currency Volatilities
GARCH volatilities can be combined with correlations computed independently, for instance from weekly returns, to produce the covariance matrix of currency factors. This approach has the advantage of combining accurate and highly responsive estimates of exchange rate volatilities with correlations computed over a longer time horizon and which are typically more robust.

**PUTTING IT TOGETHER**

Common factors, returns, exposures, and a specific risk model: everything is there except for one last critical ingredient: the covariance matrix. Building a sensible covariance matrix for more than a few factors is a complicated task that involves solving several problems.

**Coping with Incomplete Return Series**

Factor return series often have different lengths, some series starting earlier than others. Return series can also have holes. As a result, what works well for two factors is here useless. That is, filling the factor covariance matrix row i and column j using the usual formula\(^{12}\) produces a non-positive definite matrix. A statistical approach known as the EM algorithm is the conventional workaround. Details on the algorithm can be found in Dempster, Laird and Rubin,\(^{13}\) and for the purpose of this discussion, we only need to know that there exists a tool that can use incomplete series to produce an optimal estimate of the true covariance matrix.

**Global Integration**

With a model that has on the order of 180 factors, we need to solve for over 16,000 covariances. Factor returns series include, in many cases, less than 30 to 40 periods. With such a small sample size compared to the number of factors, we have a severely under-determined problem and are virtually assured that the covariance forecasts will show a large degree of spurious linear dependence among the factors. One consequence is that it becomes possible to create portfolios with artificially low risk forecasts.\(^{14}\) The structure of these portfolios would be pecu-

\(^{12}\) \(\text{Cov}(i,j) = \frac{\sum (r_i - \bar{r}_i)(r_j - \bar{r}_j)}{(N - 1)}\)


\(^{14}\) For example, by use of an optimizer.
liar—for example, they might be overweight UK AA financials, apparently hedged by an underweight in euro industrial and telecom.

Reducing the number of factors would compromise the accuracy of our risk analysis at the local level. However, we have seen for instance that the euro and sterling credit markets are to a large extent independent so that we do not need $34 \times 16/2 = 272$ covariances to describe the coupling between these two markets. Using our knowledge of the market in a more systematic fashion could go a long way in reducing the spurious correlations amongst factors.

The structured approach presented in Stefek provides a solution to this problem.\textsuperscript{15} In this method, factor returns are decomposed into a global component and a purely local component, exactly as we already decomposed asset returns into systematic and non-systematic returns. For instance, in the sterling market we can write:

$$f_{UK} = X_{fUK} \cdot g_{UK} + \varepsilon_{fUK} \quad (X.10)$$

where

- $f_{UK}$ = the vector of factor returns for the sterling market
- $g_{UK}$ = the vector of global factor returns for the sterling market
- $X_{fUK}$ = the exposure matrix of the local factors to the global factors
- $\varepsilon_{fUK}$ = the vector of residual factor returns not explained by global factors, or purely local returns.

Local factors in each market include the shift, twist, butterfly, and spread factors. Currency and emerging market factors form two independent sets of local factors. The choice of global factors is based on econometric considerations. To a large degree, sterling credit factors behave independently of factors in other markets. As a result, we know \textit{a priori} that we will gain very little by choosing more than one or two sterling global credit factors. Once the global factors are chosen, exposures are determined and also based on structural arguments.

Equation (X.9) can then be easily extended to all the original factors in the model. Assuming now that purely local returns are uncorrelated across markets and uncorrelated with global returns, the covariance matrix can be written as:

$$F = XG^TX + \Lambda \quad (X.11)$$

where

\[ G = \text{the covariance matrix of global factors} \]
\[ X = \text{the exposure matrix of the local factor to the global factors} \]
\[ \Lambda = \text{the covariance matrix of local factors} \]

Global factors could typically include:

- shift, twist, and butterfly factors (including ipb) except in euro legacy markets
- swap spread factors
- an average credit spread factor in the euro and sterling market
- an average emerging market spread
- currency factors

Far less unknowns than before now separate us from the covariance matrix. For one, there are much fewer global factors than local factors (33 against 160 if we except currencies). The local covariance matrix \( \Lambda \) is also block diagonal, with only on the order of 10,000 non-zero entries.

Unfortunately, we cannot stop there and use equation (X.11). The benefit of using global factors is that they help compute cross-market terms and constitute the skeleton of the matrix. The drawback is a loss of resolution at the local level. A solution to this problem is to replace local blocks by a local covariance matrix computed using the full set of original local factors. Off-diagonal blocks need to be adjusted in the process to insure that the final matrix is positive definite. A more detailed discussion of how the local covariance blocks are replaced can be found in Stefek. Local covariance blocks can be computed individually for each market, but also for emerging markets spread factors and currency factors. As a result, shorter half-life can be used for return series that are typically more volatile, such as currency and emerging market returns.

At this point, we have a method for building a model that reconciles two conflicting goals, that is, provide a wide coverage of markets and securities while permitting an accurate and insightful analysis, particularly at the local level.

In Exhibit XX.13, we compare correlations obtained using the standard and structured integration methods. As expected, running the EM algorithm on over 180 factors produces a large number of spurious correlations, notably between emerging market and euro spread factors. These artifacts disappear in the structured integration. Most of the cross-market coupling happens at the interest rate level, and yet, not for all markets. For instance, Czech and German Treasury yields vary in concert, but rather independently of Swiss yields. All other factors are clearly currency or market-dependent.
European fixed income portfolios are now often managed against a broad index. The risk characteristics of an index like the Merrill Lynch EMU Corporate Large Cap are presented in Exhibit X.14. This index tracks the performance of large investment grade corporate issues denominated in euro and is balanced to reflect the contributions of each market sector to the total outstanding corporate debt. Fluctuations in the euro exchange rate constitute the dominant source of risk, and would amount to about 8% per year for a US based investor. This risk disappears for investors based in the euro zone. Local market risk originates for the most part from interest rate risk, with spread risk only...
responsible for about 90 bp per year. Specific risk is small because all assets in the index are rated BBB or above.

Hedging currency and interest rate risk is relatively straightforward. This is not the case for spread risk so that understanding exposures to spread factors is a critical aspect of risk control. In Exhibit XX.15, we show a “risk map” of the Merrill Lynch EMU Corporate Large Cap index. Each row i column j entry represents the fraction of the total index return variance due to covariance between factor i and factor j. Credit factors are ordered by market sector and ratings. Lines indicate entries corresponding to negative covariances. This representation takes into account factor volatilities and correlations, the assets exposures to each factor as well as the index weight in each sector and rating category.

Because sector-by-rating spreads are relative to swaps, all assets are exposed to the swap factor, yielding a large swap risk. This risk would be transferred to the other credit factors in a model where spreads are computed relative to Treasury. The negative covariances between corporate spread returns and both the shift and swap factors can be interpreted as follows. Inspection of the correlation matrix in Exhibit XX.13 shows that correlations between credit factors and the shift factor are negative with a magnitude that tends to be small for AAA and AA like factors but can reach large values for A and BBB like factors. Swap spreads move nearly independently of corporate spreads to government. As a result, swap spreads and corporate spreads to swap are negatively correlated.
Most of the remaining credit risk originates from five clusters (C1 to C5 in Exhibit XX.15):

- A large contribution from Financial securities, which represent over 65% of the portfolio (C1).
- An unusually large contribution of Telecom assets, considering their weight in the index (~7%), that is due to very high Telecom volatilities (C2).
- A Pfandbrief “hotspot”, due to a high Pfandbrief portfolio weight of about 8% (C3).
- Two clusters of high covariances created by high Financial/Pfandbrief, Financial/Telecom and Financial/Utility cross-sector correlations (C4, C5).
Such an analysis clearly identifies risk clusters and provides important clues on how to further diversify the portfolio.

**COMPARISON WITH OTHER MARKETS**

In today’s asset management industry, organizations’ operations often extend beyond the European fixed income market. Controlling risk firm wide therefore calls for a detailed understanding of what the levels of risk in each market are and how markets interact with each other. The purpose of this last section is to provide a few elements of comparison between the euro and US dollar fixed income markets.

We compare in Exhibit XX.16 the volatilities of a few selected euro and US dollar factors. The common denominator is that euro volatilities are less than their US dollar counterparts. This is true for all factors if we ignore the volatility bursts sometimes observed over a few months for some factors (for instance the Industrial A factor in Exhibit XX.16). The average level of systematic risk observed amongst euro-denominated fixed income instruments is more generally low compared to other markets. Exhibit XX.16 shows one case where euro volatilities seem to be catching up with US levels. A more systematic analysis of how euro volatilities have recently evolved since 2002 would show that this is an exception. On average, euro volatilities have remained low with respect to US ones. Note that this is consistent first with the predictions of the swap factor model, euro spread levels and swap volatility being low compared to other markets.

Examples of correlations between shift, twist, and butterfly factors in selected markets are given in Exhibit XX.16. Not surprisingly, changes in

**EXHIBIT X.16  Selected Interest Rate Factor Correlations**

<table>
<thead>
<tr>
<th>Germany</th>
<th>Shift</th>
<th>Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.4</td>
<td>0.65</td>
</tr>
<tr>
<td>Canada</td>
<td>0.4</td>
<td>0.65</td>
</tr>
<tr>
<td>Japan</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>South Africa</td>
<td>-0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>United States</td>
<td>0.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>
European sovereign rates are generally comparable with those in other developed markets and especially the US. In 2001 and 2002, correlations have been particularly high due to the global decrease in interest rates stemming from a general economic slowdown. Exceptions correspond to emerging markets such as South Africa, and markets where the economic cycle is at a different phase, such as Japan. Conversely, high-grade credit spreads show on average very little correlation across markets. High correlations do appear, but temporarily, and for market sectors that are globally experiencing distress such as Telecommunications companies.

SUMMARY

In a complex European market, adequately measuring risk requires sophisticated methods and considerable care. A good risk model should provide a broad coverage without sacrificing accuracy, retain details but remain parsimonious, be responsive to ever-changing conditions, and so on. Certainly, there is no shortage of challenges.

The typical euro investment grade corporate index is perhaps halfway between the conservative and speculative ends of the risk spectrum. We have seen that it has very specific credit risk characteristics, such as being heavily exposed to Financials and Telecommunications. European fixed-income instruments are on average less risky than their US dollar equivalent, which by no means implies that a sound risk management is less relevant. Building a reasonable risk model is fortunately not an elusive task as long as we know how to design or where to find the right tools.

APPENDIX—PRINCIPAL COMPONENT ANALYSIS

Factor Shapes

Changes in benchmark yields for different terms are highly correlated regardless of the market, which constitutes a strong incentive to step away from a key rate model in which the factors are rate changes at the term structure vertices. The principal component analysis consists in extracting a set of linear combinations of key rate changes that capture most of the variations in a market’s benchmark. This is done mathematically.

For sovereign benchmarks, domestic government bond returns are used to compute term structures and key rate returns. For LIBOR/swap benchmarks, key rate returns can be computed directly from market yields.
ically by diagonalizing the key rate covariance matrix, each eigenvalue being a measure of how much of the benchmark variance is explained by the corresponding shape or eigenvector. The covariance matrices of principal components and key rates returns are such that:

\[ C_{PC} = T \Pi C_{KR} \Pi \]

where the columns of matrix \( \Pi \) are the principal components, and the covariance matrix \( C_{PC} \) is diagonal.

In most markets, over 95% of changes in term structures can be captured with only three principal components usually called shift, twist, and butterfly, to reflect how term structures actually change. Interest rates tend to increase or increase simultaneously, which can be described as a shift of the term structure. The second most important effect is a twist that alters the slope of the term structure. The third factor is a butterfly that reflects a change in the term structure’s curvature.

**Factor Returns**

Factor returns (hereafter called (STB) returns) are computed by regressing government bond’s returns or LIBOR/Swap key rate returns onto the shift, twist, and butterfly principal components. STB returns and other factor returns then go into the computation of the covariance matrix of all common factors. The shift, twist, and butterfly shapes are stable over time and only need to be re-estimated periodically.

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17 The diagonalization is always possible because the KR covariance matrix is symmetric.