The Barra Integrated Model

Version 204

Fati Hemmati
Agnes Hsieh
Anton V. Puchkov
Dan Stefek

September 2005
Notice and Disclaimer

Copyright © 2005 by Barra, Inc. ("Barra"). All rights reserved.

This document and all of the information contained in it, including all text, data, graphs, charts and all other information (collectively, the “Information”) may not be reproduced or redisseminated in whole or in part without prior written permission from Barra. Any use of Barra software, risk models, data or other products, services or information requires a license from Barra.

None of the Information constitutes an offer to buy or sell, or a promotion or recommendation of, any security, financial instrument or product or trading strategy, and Barra does not endorse, approve or otherwise express any opinion regarding any issuer, securities, financial products or instruments or trading strategies. Further, none of the Information is intended to constitute investment advice or a recommendation to make (or refrain from making) any kind of investment decision and may not be relied on as such.

The user of the Information assumes the entire risk of any use it may make or permit to be made of it. In particular, historical data and analysis should not be taken as an indication or guarantee of any future performance, analysis or prediction. NEITHER BARRA, ANY OF ITS AFFILIATES OR ANY OTHER THIRD PARTY INVOLVED IN MAKING OR COMPILING ANY OF THE INFORMATION MAKES ANY EXPRESS OR IMPLIED WARRANTIES OR REPRESENTATIONS WITH RESPECT TO THE INFORMATION (OR THE RESULTS TO BE OBTAINED BY THE USE THEREOF), AND BARRA, ITS AFFILIATES AND EACH SUCH OTHER THIRD PARTY HEREBY EXPRESSLY DISCLAIM ALL IMPLIED WARRANTIES (INCLUDING, WITHOUT LIMITATION, ANY IMPLIED WARRANTIES OF ORIGINALITY, ACCURACY, TIMELINESS, NON-INFRINGEMENT, COMPLETENESS, MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE) WITH RESPECT TO ANY OF THE INFORMATION.

Without limiting any of the foregoing, in no event shall Barra, any of its affiliates or any other third party involved in making or compiling any of the Information have any liability regarding any of the Information for any direct, indirect, special, punitive, consequential or any other damages (including lost profits) even if notified of the possibility of such damages.

Barra® and all other service marks referred to herein are the exclusive property of Barra or its affiliates. All Barra software, risk models, data or other products, services or information are the exclusive property of Barra and/or its third party vendors and may not be used in any way without the express written permission of Barra.
The Barra Integrated Model (BIM) is a multi-asset class model for forecasting the asset and portfolio level risk of global equities, bonds, currencies, and commodities. The model uses innovative methods to couple broad asset coverage with the detailed analysis of Barra’s models that focus on particular markets. This makes it suitable for a wide range of investment purposes, from conducting an in-depth analysis of a single-country portfolio to understanding the risk profile of a broad set of international investments.

This paper describes the methodology and construction of the model. 1 We give an overview of our aims and approach in Section 1. In Section 2 we describe the common basic structure we use to build the global asset-class models. Sections 3 and 4 look at the application of this structure to equities and bonds. Sections 5 and 6 describe how currencies and commodities are modeled and Section 7 details how we combine all the components to generate the full Barra Integrated Model.

1 Overview

Our chief goal is to provide a single model that best forecasts the risk of a wide range of portfolios, from those concentrated in a single market to large, international multi-asset class portfolios. To do this, the model must offer both broad coverage and in-depth analysis. It should be detailed enough to allow a manager to drill down to his assets in a local market and obtain an insightful and accurate analysis, and yet broad enough to cover a large investment universe. Unfortunately, these objectives are conflicting. As new markets are added to a global model, the complexity needed to maintain a fine level of detail increases, posing a serious econometric challenge. Until now, this goal has been elusive.

In the Barra Integrated Model, we employ a novel methodology to achieve this goal. First, to provide the needed level of detail, we build factor models of all the local equity and bond markets. These models attribute the explainable portion of an asset’s return to the local factors at work in each market. These factors include styles and industries for equities, and term structure movements and credit spreads for bonds. They may differ significantly from market to market. By modeling each market individually, we enable investors to see their exposures to various factors of each particular market and also provide the most accurate forecasts of local market risk.

1 Numerous other individuals contributed to this paper, notably Neil Gilfedder.
Next, we build two asset class models, one for equities and one for bonds, by combining the local models in each class, ignoring currencies for the moment. This requires an understanding of what links the behavior of assets across markets. Since asset returns are driven, in part, by local factors, the key to developing each model is to determine the covariance between these factors across different markets. There are far too many factors to reliably estimate their covariances directly on the basis of available data. Fortunately, within each asset class, a much smaller set of inter-market or “global” \(^2\) factors accounts for much of the cross-market correlation. By building structural models of how these global factors link local factors across markets, we are able to obtain estimates that are more accurate.

Our use of structural models provides a new framework for global analysis. These models decompose local factor returns into a part due to global factors (that is shared across markets) and a part that is purely local (that only affects the securities within each market). This explains, for example, why the industry risk of a US bank is better hedged with another US bank than with a Japanese bank.

We complete the single asset-class models by adding currency and commodity models. Our final step is then to combine all models together to form the complete Barra Integrated Model. Leveraging our earlier work, we use the global factors to estimate the correlation structure across asset classes.

2 Building Global Asset Class Models

In this section, we describe the general framework for constructing single asset-class models for global equities and global bonds, paying special attention to the modeling aspects that are common to both asset classes. In later sections, we describe the details of our approach that are specific to each asset class.

Our approach to modeling global equities and bonds consists of building a set of local risk models and establishing links between them.\(^3\) It is based on the view that an asset’s return is strongly influenced by local market factors. These factors may differ in number, character and behavior across markets. For example, the US equity market is well characterized by 13 style

\(^2\) The term “global” simply refers to the role of these factors in determining cross-market covariances.

\(^3\) Examples of local models are the Australia Equity Model, Japan Bond Model, etc. While single-country equity or bond models are possible choices for local models, they are not the only possible local models. For example, the Europe Equity Model, a regional model that covers securities listed in 16 Western European markets, would also qualify as a local model.
factors and 55 industries with significant concentration in technology, finance and health care. In contrast, the Australian equity market can be captured with fewer factors and has more exposure to basic materials. The bond models are more uniform in structure, each incorporating three factors to account for interest rate (term structure) movements, and one or more credit spread factors.

2.1 The Structure of Local Models

The local models are the building blocks of our global model. Each local model decomposes an asset’s local excess return into a part due to local factors and a part that is unique to the underlying asset, the specific return. Using Australian equities as an example, this is written mathematically as:

\[ r_{\text{aus}} = X_{\text{aus}}^t f_{\text{aus}} + u_{\text{aus}} \]  

where \( r \) is a vector of excess returns, \( X \) is a matrix of asset exposures to the common factors, \( f \) is a vector of factor returns and \( u \) is a vector of specific returns. It is assumed that \( f \) and \( u \) are uncorrelated and that the \( u \)'s are uncorrelated across different assets.

We form a factor covariance matrix, \( F_{\text{aus}} \), using exponentially declining weights for the historical factor returns. We also form a diagonal matrix of asset specific variances, \( \Delta_{\text{aus}} \).

To compute the risk of a portfolio of Australian equities, we need a forecast covariance matrix of Australian asset returns, \( V_{\text{aus}} \). Using our factor covariance matrix, \( F_{\text{aus}} \) and our matrix of asset specific variances, \( \Delta_{\text{aus}} \):

\[ V_{\text{aus}} = X_{\text{aus}}^t F_{\text{aus}} X_{\text{aus}}^t + \Delta_{\text{aus}} \]

---

4 For full details on local models, see the appropriate handbook, e.g. United States Equity Risk Model Handbook.

5 For equities, the excess return of an asset is \( r - r_f \) where \( r \) is the asset's total return and \( r_f \) is the risk-free rate. For bonds, excess returns are computed as \( \frac{P_{t+1} - F_{t+1}}{P_t} \) where \( P_{t+1} \) is the price at time \( t+1 \), \( P_t \) is the price at time \( t \), and \( F_{t+1} \) is the forward price for time \( t+1 \) computed at time \( t \). \( F_{t+1} \) is computed using a valuation model.
Thus, the risk of a portfolio arises from its exposure to factors in the market as well as from the idiosyncratic behavior of individual securities it contains.

2.2 Aggregating Local Models

To compute the risk of a portfolio of international equities (or international bonds), we need a global equity (or bond) risk model that gives the covariance between returns to equities (or bonds) in different markets. We form such a model by aggregating our local models. The factors of this new model are simply all the local market factors. Further, each asset is exposed only to its own market’s factors. Using global equities as an example, the factor exposure, $X_E$, factor covariance, $F_E$, and specific variance, $D_E$, matrices of this model can be written as:

$$
X_E = \begin{pmatrix}
X_{aus} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & X_{usa}
\end{pmatrix}
$$

$$
F_E = \begin{pmatrix}
F_{aus} & \cdots & F_{aus,usa} \\
\vdots & \ddots & \vdots \\
F_{usa,aus} & \cdots & F_{usa}
\end{pmatrix}
$$

$$
\Delta_E = \begin{pmatrix}
\Delta_{aus} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \Delta_{usa}
\end{pmatrix}
$$

(3)

Using our new model, we can write the global equity asset covariance matrix in the familiar form:

$$
V_E = X_E F_E X_E^T + \Delta_E
$$

(4)

In a similar manner, we can write the global for other asset classes.

2.3 Modeling Covariances of Local Factors in Different Markets

Continuing our global equity example, we now need to more fully specify the factor covariance matrices $F_E$. The diagonal blocks of this matrix contain covariances between the local factors within each market; the local models have already provided these. What remains to be specified is the covariance between local factors in different markets. This involves estimating a significant number of covariances, since there are a large number of local equity factors. The underlying data are available at monthly intervals and in many cases go back fewer than 15 years. Trying to compute these covariances directly will almost certainly result in numerous spurious relationships.
Our solution to this problem is to use a *structural model* for each asset class that establishes a sensible relationship between factors in different markets. The idea is that the behavior of local factors may be accounted for, in part, by a much smaller set of *global* factors. For example, part of the return to the local US and UK oil factors is due to an underlying global oil factor, which captures global oil prices, cartel activity, etc. In similar fashion, spreads on corporate bonds of different credit qualities in the US and UK are partly driven by corporate spread factors for these countries. These global factors link local factors across markets, accounting for any correlation between them. To estimate the covariance between local market factors, we only need to determine a much smaller set of global factor covariances, improving the reliability of our estimates.

For any asset class covered by BIM the structural model has the following form:

\[
    f_{ac} = Y_{ac} g_{ac} + \phi_{ac} 
\]

where

- \(ac\) = asset class (e.g. equity, bonds, etc)
- \(f_{ac}\) = vector of local asset class (e.g. equity, bond, etc) factor returns across all markets
- \(Y_{ac}\) = a matrix of exposures of the local factors to the global factors
- \(g_{ac}\) = a vector of global factor returns for the asset class
- \(\phi_{ac}\) = a vector of the purely local factor returns.\(^6\)

The structural models not only overcome the econometric problem, but also provide a new framework for global analysis. They decompose each local factor return into a part due to global factors and a part that is *purely local*. These purely local returns are not correlated across markets but may be correlated within each market. This construction allows the factors in different markets to be correlated but not identical.

From the structural models, we obtain an estimate of the asset class factor covariance matrix, \(\hat{F}_{ac}\), consisting of two parts:

\[
    \hat{F}_{ac} = \underbrace{Y_{ac} G_{ac} Y_{ac}'}_{\text{Due to global factors}} + \underbrace{\Phi_{ac}}_{\text{Due to purely local factors}} 
\]

\(^6\) There are differences in implementation of the global factor models for equities and bonds, such as the set of global factors used, the calculation of local factor exposures to global factors, etc. In later sections, we discuss the implementation details for each asset class.
where
\[ G_{ac} = \text{covariance matrix of global factors} \]
\[ \Phi_{ac} = \text{covariance matrix of purely local factors with } \Phi_{ac,ij} = 0 \text{ if } i \text{ and } j \text{ are not in the same market} \]

2.4 Consistency Between Local Models and Global Model

One of our aims is that our forecasts of local risk be consistent with those of the underlying local model. In building \( \hat{F}_{ac} \), we simply sought good estimates of the covariance between factors in different markets. We intended to use the factor covariance matrices from the local models as the diagonal blocks of \( F_{ac} \) since these are our best estimates. Given that the diagonal blocks of \( \hat{F}_{ac} \) generally differ from our target, our last step is to form the final covariance matrix of all the local factors, \( F_{ac} \) by altering \( \hat{F}_{ac} \) in Equation (7) so that the local blocks are consistent with the local models. (So, for example, the final covariances of local factors for the Australian market are the same as those in the Australia Equity Model). We do this by scaling the local covariance matrices into \( \hat{F}_{ac} \) as described in Appendix A.

2.5 Historical changes in BIM factor list.

Going back in history, BIM has a very fluid local factor list. Some local models did not exist five years ago, while others had a different set of local factors. Similarly, European currencies ceased to exist as independent assets in the end of the last century with the advent of the Euro. Overall, the list of local factors in BIM grew from less than 1200 in the end of 1997 to more than 1500 in August 2005. Obviously, this affects a list of global factors as well. References to a number of factors found in the remainder of this document, including Appendix C, refer to the model structure as of August 2005.

3 Global Equities

Our new approach to modeling global equities differs markedly from that of most global equity models that use a single set of factors to characterize the risk of equities throughout the world.\(^7\)

\(^7\) For example, Barra’s Global Equity Model (GEM).

© MSCIBarra, 2005
Those models assume that all equities are driven by exactly one parsimonious set of factors, implying that returns due to industries and styles move in lockstep across markets.

<table>
<thead>
<tr>
<th>Local Equity Factor</th>
<th>Average Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>.44</td>
</tr>
<tr>
<td>Finance</td>
<td>.46</td>
</tr>
<tr>
<td>Information Technology</td>
<td>.42</td>
</tr>
<tr>
<td>Momentum</td>
<td>.36</td>
</tr>
<tr>
<td>Size</td>
<td>.27</td>
</tr>
<tr>
<td>Volatility</td>
<td>.42</td>
</tr>
</tbody>
</table>

Table 1 presents evidence supporting our view. The table shows the average pair-wise correlation between the same factors (e.g. US size, UK size) in nine major markets. We find that there is some commonality in the behavior of both industries and risk indices across countries. Despite this correlation, it is clear that these factors behave significantly differently from market to market.

### 3.1 Global Equity Factors

The global factors for which we have chosen to explain the covariances of equity factors across local models are:

1. A world factor
2. Country factors
3. Global industry factors
4. Global style factors: Price Momentum, Size, Value and Volatility

The world factor captures the global market return, while the industry and country factors reflect the return to global industry and country influences net of other factors. Both the industry factors and the country factors render the world factor redundant, creating what is known as an identification problem. To resolve this, we follow the standard practice of requiring both a weighted combination of the countries and of the industry factor returns to sum to zero.

---

8 The nine markets are: Australia, Canada, France, Germany, Italy, Japan, Switzerland, UK and US.

9 A list of global factors is available in Appendix C.

10 Both the industry factors and the country factors render the world factor redundant, creating what is known as an identification problem. To resolve this, we follow the standard practice of requiring both a weighted combination of the countries and of the industry factor returns to sum to zero.
3.2 Exposures of Local Equity Factors to Global Equity Factors

We pre-specify the exposures of local equity factors to global factors as follows. Each local industry factor has unit exposure to the world factor, $g_{wld}$, its own country factor, $g_{cnty}$, and the global industry to which it belongs, $g_{ind}$. It has no other global exposures. Each local style factor (or risk index) corresponding to one of the four global styles has unit exposure to that style, $g_{ri}$, and no exposure to other factors. The other local styles have no global exposure.

$$f_{ind} = g_{wld} + Y_{cnty}g_{cnty} + Y_{ind}g_{ind} + \phi_{ind} \quad (7)$$

$$f_{ri} = g_{ri} + \phi_{ri} \quad (8)$$

3.3 Estimating Returns to Global Equity Factors

We compiled a history of returns to global equity factors by fitting the structural model in Equations (8) and (9) to monthly local factor returns. The global factor returns were estimated using cross sectional weighted least squares regression subject to constraints. The time period covered by this estimation was January 1984 to the present. Many of the country factor returns had incomplete histories because models for these markets did not exist until some time after 1984. Where possible, we constructed proxies for these missing returns using country index returns.

3.4 Computing Covariances of Global Equity Factors

We computed the covariance matrix of the global equity factors, the $G_{ac}$, in Equation (7) from historical estimates of the returns to these factors. To account for changing relationships among these factors, we placed greater weight on more recent returns when constructing the covariance matrix. In particular, we used an exponentially weighted scheme with a half-life of 48 months, in other words, weighting this month’s factor return twice as much as one four years ago. To cope with missing data, we used the Expectation Maximization (EM) algorithm. The

---

11 A local industry factor may, in some cases, be exposed to more than one global industry factor with fractional weights in each.

12 Each local industry factor’s weight was roughly the sum of the square roots of the capitalizations of the assets in the estimation universe exposed to that factor. Each local style factor was given a weight proportional to the square root of capitalization of the assets in the estimation universe of the corresponding local model.

13 See footnote 10 for a description of the constraints.

14 The EM algorithm is used to calculate a best covariance matrix, in a maximum likelihood sense, when some of the factors have incomplete histories. See Dempster, Laird and Rubin, “Maximum Likelihood
covariance matrix of the purely local part of equity factor returns, the $\Phi_{ac}$ in Equation (7), was computed in a similar manner using a half-life of 48 months.

The final step is to apply the procedure described in Appendix A to obtain a block-diagonal scaling matrix that is required in order to make the covariance block for each local market match that of the corresponding local model.

### 4 Global Bonds

Our approach to building a global bond model parallels our work on equities.\(^{15}\) We start by building factor models for each of the local bond markets. The factors at work in these local markets include shift, twist and butterfly term structure movements (STB) and a swap spread. Six markets — Canada, Japan, Switzerland, the US, the UK, and the euro-zone — have more detailed credit factors that explain the spread over swap on the basis of sector, or sector by rating classifications.\(^{16}\) Additionally, emerging market bonds denominated in an external currency have country dependent spreads — one spread for all bonds from each emerging market — that enable us to include them in our model. Altogether, there are over 400 local factors.

#### 4.1 Global Bond Factors

In Section 3, we described how we build the factor model that links local equity models by pre-specifying local equity factors’ exposures to global factors and estimating the returns to these global factors. In contrast, our approach in building the factor model that links local bond models is to pre-specify the returns to global factors and estimate the exposures of each local bond factor to these global factors through time-series regressions. The risk of US municipal bonds (“munis”) is captured in the local US bond model using a separate yield curve characterized by muni STB factors. Municipal bonds may additionally be exposed to a credit spread factor. In this section, we describe the global factors we use, and how their returns are calculated.

---

\(^{15}\) For further details, see O. Cheyette, “Global Credit Risk Modeling,” Barra Research Insights, 2002.

\(^{16}\) For the purposes of modeling corporate bonds, we have a single euro-credit model that spans twelve markets. The structure of this model is similar to that of the other markets, so it makes no real difference for our exposition whether we consider the euro-zone to be one market or twelve individual markets — we will think of it as one.
The global factors that we have included in the model to capture covariances in bond factors across different local markets are:

1. The shift and twist factors from each of the local markets
2. The swap spread factor from each local market, where available
3. An average credit spread factor for each of the following: Canada, Japan, Switzerland, US, UK, and euro-zone
4. An average emerging market credit spread factor
5. US muni shift and twist factors
6. Implied volatility factors for each of Japan, UK, US and the euro-zone
7. Shift factors for real interest rates in the US, UK, Canada and the euro-zone

Note that some factors, e.g. the local shift, twist and swap spreads, are themselves global factors as well. This simply means that we do not use any other factors as proxies for them in estimating their covariance with the other local factors. We decided to do this because some of these variables are significantly correlated across markets and the gain from proxying them with a reduced set of variables was small.

Table 2 provides some support for this decision, giving the average pair-wise correlations between shift, twist and swap factors across markets (thus, it looks at all shift pairs,

<table>
<thead>
<tr>
<th>Period</th>
<th>Shift</th>
<th>Twist</th>
<th>Swap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 1993 to Aug 2005</td>
<td>.49</td>
<td>.26</td>
<td>.15</td>
</tr>
<tr>
<td>Average Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Significantly Positive</td>
<td>89%</td>
<td>69%</td>
<td>48%</td>
</tr>
<tr>
<td>Jan 1993 to Dec 1996</td>
<td>.45</td>
<td>.15</td>
<td>.15</td>
</tr>
<tr>
<td>Average Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Significantly Positive</td>
<td>81%</td>
<td>25%</td>
<td>24%</td>
</tr>
<tr>
<td>Jan 1997 to Dec 2002</td>
<td>.50</td>
<td>.38</td>
<td>.16</td>
</tr>
<tr>
<td>Average Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Significantly Positive</td>
<td>78%</td>
<td>72%</td>
<td>33%</td>
</tr>
<tr>
<td>Jan 2003 to Aug 2005</td>
<td>.70</td>
<td>.26</td>
<td>.11</td>
</tr>
<tr>
<td>Average Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Significantly Positive</td>
<td>83%</td>
<td>33%</td>
<td>19%</td>
</tr>
</tbody>
</table>

See the Appendix C for a list of global bond factors.
twist pairs, etc.) and the percentage of significant positive correlations over different time periods. Clearly, shift and twist and to some extent swaps are correlated significantly across markets.

Turning to credit spreads, we find that these factors are strongly correlated within each of the US, UK, Japan and the euro-zone markets. We illustrate the magnitude of this correlation in Tables 3A and 3B using the UK and euro-zone credit spreads, respectively.

We capture this commonality in an average spread factor and use it to help account for the correlation of local credit spreads with factors in other markets. The return to this factor is defined as:

\[
\text{Average credit spread} = \sum_k w_k f_k
\]  

(9)

where \( k \in \) local credit factors and each factor’s weight, \( w_k \), is inversely proportional to its volatility. We selected these weights, in part, to mitigate the influence of the lower quality credit factors that tend to have substantially higher volatilities.

We use emerging market bond spreads as additional factors to explain the risk of emerging market debt issues denominated in external currencies. For integrating these factors into the BIM covariance matrix, we divide them into two groups based on the availability of a local-currency yield curve model. For the emerging markets where the local models are available, we use global factors from these models to integrate the external-currency denominated spreads. For the markets where a local-currency yield curve model is not available, we notice these spreads to be strongly correlated. Over the period from January 1998 to August 2005, the

<table>
<thead>
<tr>
<th>TABLE 3A</th>
<th>Average Correlation of UK Credit Spread Factors, May 1999 to Aug 2005(^{18})</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>.81</td>
</tr>
<tr>
<td>AA</td>
<td>.69</td>
</tr>
<tr>
<td>A</td>
<td>.60</td>
</tr>
<tr>
<td>BBB</td>
<td>.50 AAA</td>
</tr>
<tr>
<td></td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td>.76</td>
</tr>
<tr>
<td></td>
<td>.65 AA</td>
</tr>
<tr>
<td></td>
<td>.85</td>
</tr>
<tr>
<td></td>
<td>.75 A</td>
</tr>
<tr>
<td></td>
<td>.70 BBB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 3B</th>
<th>Average Correlation of Euro Zone Credit Spread Factors, June 1999 to Aug 2005(^{19})</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>.81</td>
</tr>
<tr>
<td>AA</td>
<td>.65</td>
</tr>
<tr>
<td>A</td>
<td>.49</td>
</tr>
<tr>
<td>BBB</td>
<td>.19 AAA</td>
</tr>
<tr>
<td></td>
<td>.60</td>
</tr>
<tr>
<td></td>
<td>.55</td>
</tr>
<tr>
<td></td>
<td>.34 AA</td>
</tr>
<tr>
<td></td>
<td>.66</td>
</tr>
<tr>
<td></td>
<td>.58 A</td>
</tr>
<tr>
<td></td>
<td>.71 BBB</td>
</tr>
</tbody>
</table>

\(^{18}\) Each cell in the table is the average correlation of spread factors, either for a single rating category or between two rating categories, but across different sectors. This is why the diagonals, the average correlation within a rating category, do not equal one.
average correlation was 0.35. Therefore, as in the case of the spread factors in UK, US, Euro, Japan, Canada and Switzerland, we formed an average emerging market credit factor that reflects the common behavior of these markets, defining it to be:

\[
\text{Average emerging market credit spread} = \sum_k w_k f_k
\]  

(10)

Here, \(k\) runs over emerging market credit factors that do not have a local-currency yield curve model, and the weight on each factor is inversely proportional to its volatility.

### 4.2 Exposures of Local Bond Factors to Global Bond Factors

The exposures of local bond factors to the global bond factors are defined as follows:

- The local shift, twist and swap factors each have unit exposure to the corresponding global factors.
- The local credit spread factors in the US, UK, Japan, Canada, Switzerland, and euro markets are exposed to the average credit and global swap factors corresponding to their market. The exposures are the regression coefficients obtained from regressing the time series of local credit spread factors simultaneously on the average credit and global swap factors.
- The local emerging market credit spread factors, denominated in external currency, in markets that do not have a local-currency yield curve model are exposed to the average emerging market credit spread factor. The exposures are the regression coefficients obtained from regressing the time series of the local factors returns on this global factor.
- The local emerging market credit spread factors, denominated in external currency, in markets that do have a local-currency yield curve model are exposed to that model’s global yield curve shift factor and, if available, global swap factor. The exposures are the regression coefficients obtained from regressing the time series of the local factors returns on the global factors.
- The local US MUNI shift and twist factors each have unit exposure to the corresponding global factors.

### 4.3 Computing Covariance Matrix Of Global Bond Factors

As with equities, we need to estimate \(G_{ac}\) and \(\Phi_{ac}\) for bonds. We estimate these from the global and purely local factor returns using an exponential weighting scheme with a half-life of 24 months. We use the EM algorithm to cope with missing data.

\[\text{19 Omitting the month surrounding the Russian default, August 1998, reduces this correlation to 0.26, which is still substantial.}\]
The last step is to calculate a block-diagonal scaling matrix required to adjust block diagonals of the forecast covariance matrix given by Equation (6) to be in agreement with the single-country model covariance matrices build independently.

5 The Currency Model

The global investor is interested in the risk of and return to his portfolio from a particular currency or numeraire perspective. For modeling purposes, we decompose the excess return in the numeraire currency into a part due to currency fluctuations and a part due to local equity. Consider the excess return from a US dollar perspective of an investment in Sony Corp on the Tokyo Stock Exchange, $r_{Sony/S}$. We can write this as:

$$r_{Sony/S} - rf_{usa} = (1 + ex_{y/S}) (1 + r_{Sony}) - (1 + rf_{usa})$$

where

- $r_{Sony} =$ local return to Sony
- $rf_{usa} = $ US risk free rate
- $rf_{jpn} = $ Japanese risk free
- $ex_{y/S} =$ exchange return to an investment in yen from a dollar perspective

We define the currency return to be the excess return to an investment in a foreign instrument yielding the short-term rate.

We regard the currency returns as local factors. Cash holdings have unit exposure to the appropriate currency factor. Most currencies, with the notable exception of European currencies, are treated as both local (to the currency asset class) and global factors (like the bond term structure factors). For these currencies, we can formally write:

$$f_C = Y_C g_C$$

Here, the purely local return is zero. The list of global currency factors can be found in Table 2.

European currencies are tightly related to Euro. Indeed, in a period 1/2000 to 8/2005 the average correlation of excess return to 12 European currencies, marked with $M_e$ in Table 2, with that of Euro is 74%. Therefore, for the European region currencies we separate return into Euro-related and idiosyncratic. Formally, we build a single-factor European currency sub-model:

$$r_{cur} = \beta r_e + \phi_{cur}$$
Here, $r$ is a vector of returns to European currencies covered by the sub-model, $r_€$ is Euro currency return, and $\beta$ is calculated using historical return data with half-life of 24 months. The list of currencies covered by the European sub-model can be found in Table 2.

Euro currency is a global factor while currencies described by Equation (13) are local. For all currencies we can write:

$$f_c = Y_c g_c + \phi_c$$

(14)

Here, loadings $Y_c$ are different from unitary exposure to itself only for the dependent factors in the European sub-model. The same currencies have $\phi_c$ different from zero.

The covariance matrix $F_c$ is given by Equation (6) where $G_{ac}$ is calculated using global currency returns $g_c$ and EM algorithm with 24-month half-life. The covariance of purely local returns is calculated similarly using the residual returns for the European sub-model currencies. For the rest of the currencies purely local variances are zero.

We also construct the local currency covariance matrix separately. Here, currency volatilities are estimated from daily returns using GARCH (1,1) models while correlations are estimated from weekly data using an exponential weighting scheme with a 17-weeks half-life. Finally, we apply the procedure described in Appendix A to obtain a block-diagonal scaling matrix that is required in order to make $F_c$ calculated in the previous step to match the separately estimated currency covariance.

6 The Commodity Model

We use Goldman Sachs Commodity Total Return Indices (GSCI-TR), that measure the returns accrued from investing in fully-collateralized nearby commodity futures, as local factors. Currently, BIM covers 25 sub-sector commodity indices from all commodity sectors: six energy indices, five industrial metals, eight agricultural, three livestock and three precious metals. Although there are enough observations to estimate the covariance matrix of the 25 indices

---

using historical data, we need to build a global factor model to integrate GSCI indices into the BIM covariance matrix. Our model is of the following form:

\[ f_M = \sum_{i=1}^{K} \beta_i g_i + \phi_M \]  

(15)

Here, \( f_M \) is a vector of returns to GSCI sub-sector commodity indices, \( g_i \)'s are returns to global factors, \( \beta \)'s are factor loadings and \( \phi_M \) is a vector of idiosyncratic returns. The global factors in our model are the five aggregated GSCI sector indices provided by Goldman Sachs and listed in Table 4. The loadings \( \beta \) are calculated using historical returns and a half-life of 24 months. Each local commodity factor (e.g. GSCI sub-sector) is exposed to a single global factor (e.g. GSCI sector), that it is part of. For example, GSCI Soybeans factor is exposed only to GSCI Agriculture. Thus, we separate return to a sub-sector commodity index into a part related to commodity sector return, and an idiosyncratic part.

The covariance matrix is given by:

\[ F_M = Y_M^t G_M Y_M + \Phi_M \]  

(16)

where matrix of exposures \( Y_M \) is given by \( \beta \)'s estimated from Equation (15). Finally, we calculate the scaling matrix required to match covariance matrix (16) to that estimated on historical returns of GSCI sub-sector indices using EM algorithm and 24-month half-life.

While the diagonal commodity block is estimated historically, according to Equation (6) correlation of commodity factors with the rest of the factors in BIM covariance matrix will be driven by the correlation of commodity sectors and global factors from other asset classes.

7 Putting It All Together – A Multi-Asset Class Risk Model

We form the Barra Integrated Model by combining the individual asset class models. The Barra Integrated Model’s factors include all the local equity, bond, currency and commodity factors. The exposure matrix, \( X_{BIM} \) and the specific risk matrix, \( \Delta_{BIM} \) are of the form:
where $X_{E,C}$, $X_{B,C}$ and $X_C$ are the exposures of equities, bonds and currencies to the currency factors\(^{21}\), and $X_M$ is an exposure matrix of commodities to the GSCI Index factors (in USD). From the decomposition in Equation (12), it is clear that each equity and bond has a unit exposure to the currency of its own market and no exposure to any other currency.

To complete our multi-asset class model, we must specify the covariance between factors in different asset classes. Drawing on our earlier work, the natural answer is that these factors are related through the global factors in each asset class. These global factors embody the information that relates markets within an asset class and are therefore likely to capture important links across asset classes. This implies that the Barra Integrated Model factor covariance matrix is:

\[
F_{BM} = \begin{pmatrix}
Y_E & 0 & 0 & 0 \\
0 & Y_C & 0 & 0 \\
0 & 0 & Y_B & 0 \\
0 & 0 & 0 & Y_M
\end{pmatrix} \begin{pmatrix}
G_E & G_{E,C} & G_{E,B} & G_{E,M} \\
G_{C,E} & G_C & G_{C,B} & G_{C,M} \\
G_{B,E} & G_{B,C} & G_B & G_{B,M} \\
G_{M,E} & G_{M,C} & G_{M,B} & G_M
\end{pmatrix} \begin{pmatrix}
Y_E & 0 & 0 & 0 \\
0 & Y_C & 0 & 0 \\
0 & 0 & Y_B & 0 \\
0 & 0 & 0 & Y_M
\end{pmatrix}^t
\]

\[
Y_E = 0 = Y_C = Y_B = 0
\]

\[
0 = 0 = 0 = 0
\]

where the notation $G_{X,Y}$ denotes the covariance between the global factors of asset classes $X$ and $Y$.

\[\text{Equation (18)}\]

\[\text{Equation (19)}\]

\(\Delta_{BM} = \begin{pmatrix}
\Delta_E & 0 & 0 & 0 \\
0 & \Delta_C & 0 & 0 \\
0 & 0 & \Delta_E & 0 \\
0 & 0 & 0 & \Delta_M
\end{pmatrix}\]

\[\text{Equation (17)}\]

\(\text{Equation (12)}\)

\[\Phi_E = 0 \quad 0 \quad 0 \quad 0
\]

\[\Phi_C = 0 \quad 0 \quad 0
\]

\[\Phi_B = 0 \quad 0
\]

\[\Phi_M = 0 \quad 0
\]

\[\text{Equation (18)}\]

\[\text{Equation (19)}\]

---

\(^{21}\) More precisely, an investment in a bond or equity in a local market incurs an implicit exposure to the currency of that market.
How significant are the links between asset classes? Table 4 shows the relationship between (global) equity and bond factors in the same market. For each market, we computed the correlation between country returns, on the one hand, and term structure factor returns and the average emerging market credit factor returns, on the other.24

Table 4 presents the average correlation across markets. Among the term structure factors, shifts are most significantly correlated with the equity factor. For emerging markets, there is a very strong correlation between the average emerging market credit factor and the country factor.

With more than 220 global factors, there are too many to reliably estimate their correlations directly, given the amount of historical data available and the short histories of some of the factors. To overcome this, we designated a subset of the global factors, those most likely to account for correlations between asset classes, as core factors. These are:

1. World factor
2. Country equity factors
3. Shift factors for major markets
4. Swap spread factors for major markets
5. Major currencies25

### Table 4

<table>
<thead>
<tr>
<th>Shift</th>
<th>Twist</th>
<th>Swap</th>
<th>Ave Emerging Market Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Correlation</td>
<td>.09</td>
<td>.03</td>
<td>.07</td>
</tr>
<tr>
<td>% Positive and Significant</td>
<td>40%</td>
<td>30%</td>
<td>22%</td>
</tr>
<tr>
<td>% Negative and Significant</td>
<td>18%</td>
<td>9%</td>
<td>6%</td>
</tr>
</tbody>
</table>

22 Global equity factors are the country equity factors plus the World factor, and global bond factors are Shift, Twist, Swap and Average Emerging Market Credit Spread.

23 Average emerging market credit spread begins January 1998.

24 For our purposes, we define the country return to be the country factor return plus the World factor return. The correlation between our measure and the returns to country indices for most countries is greater than 0.9.

25 The list of core factor can be found in Appendix C.
As an approximation, we assume that the correlations between global factors in different asset classes would be expressed through these core factors. More precisely, we posit that for any set of global factors $A$

$$g_A = \alpha_A + \sum_{A,\text{core}} \Sigma_{A,\text{core}}^{-1} g_{\text{core}} + \tau_A$$  

(19)

where

- $g_A$ = a vector of returns to the global factors in $A$
- $g_{\text{core}}$ = a vector of returns of the core factors
- $\Sigma_{A,\text{core}}$ = a matrix of covariances between the global factors in $A$ and the core factors
- $\Sigma_{\text{core}}$ = a matrix of the variances and covariances of the core factors

The components of $\tau_A$, a vector of the returns of the global factors residual to the core factor returns, are assumed to be uncorrelated with the $g$'s. We also assume that $\tau_{Ak,p}$ and $\tau_{Ak,q}$ are uncorrelated with each other when $p$ and $q$ index global factors are corresponding to different asset classes.

We can then express the covariances between any two sets of global factors $A$ and $B$ as:

$$G_{A,B} = \Sigma_{A,\text{core}}^{-1} \Sigma_{\text{core},B} + \sum_{A,B}$$

(20)

The covariance matrix of residuals is computed as conditional variance of global factors $A$ given the covariance matrix of core factors:

$$\Sigma_{A,\text{core}}^{-1} \Sigma_{\text{core},A} = \Sigma_A - \Sigma_{A,\text{core}}^{-1} \Sigma_{\text{core},A}$$

$$\sum_{A,B} = 0, \quad \text{if} \quad \tau_A \neq \tau_B$$  

(21)

Notice that the second term in the first of Equations (21), and the first term in Equation (20) do not necessarily cancel out since they are estimated using EM algorithm on different sets of factors. Also, $\Sigma_{A,\text{core}}$ given by Equation (21) is positive (semi)definite. We use formulae (20) and (21) to compute to a provisional estimate $G_{BIM}$.26

26 The covariance matrices $\Sigma_{A,\text{core}}$ and $\Sigma_{\text{core}}$ are estimated from the global factors using an exponentially weighted scheme with a half-life of 24 months.
To ensure consistency with our asset class models, we scale in the covariance matrices for global bond, equity, currency and commodity factors using the same scaling procedure that we used for individual asset classes.

With $G_{BIM}$ known, the full local factor level covariance matrix is calculated using Equation (18) with $Y$ and $\Phi$ taken from the asset class models. In a final step, we use the block-diagonal scaling matrix, combined diagonally from the asset class scaling matrices calculated above, to bring the diagonal blocks of covariance matrix (18) into agreement with the separately estimated local models. The technical details of this procedure are described in Appendix A.

8 Summary

The Barra Integrated Model is a multi-asset class risk model that couples breadth of coverage (global equities, global bonds, currencies and commodities) with the depth of analysis provided by our local models. Users no longer have to choose between granularity of local model analysis, on the one hand, and the broad scope of global model analysis, on the other. The model is suitable for a wide range of investment needs, from analysis of a single-country equity portfolio to a plan-wide international portfolio of equities and bonds.

To provide both depth and breadth of analysis, we have developed a new approach to modeling global risk. In-depth, accurate local analysis requires that we choose factors that are effective in the market under study and that we recognize that the factors developed for one market are not always appropriate for use in other markets. Thus, we start by building individual risk models for each market to best capture the behavior of the local securities.

For broad global analysis, we must determine how securities in different markets co-vary. We accomplish this by modeling the relationships between the factors across local markets. The number of correlations between factors rises sharply with the number of factors. There is simply not enough data to estimate so many correlations directly. Fortunately, these relationships may be modeled using a smaller set of global factors. Correlations across markets are expressed through these global factors, requiring less data for accurate estimation.

The model is also flexible in its structure, allowing us to incorporate advances in modeling for different countries or regions. For example, if we find that a group of countries is better

---

27 The number of correlations is $\frac{N(N-1)}{2}$ where $N$ is the number of factors.
modeled as a bloc, then a local model for the bloc may replace local models for the individual countries. Its design also enables us to add additional asset classes (e.g. real estate or hedge funds) without changing the model’s architecture.

Appendix A

Consistency Between Local Models and the Global Model

In this appendix, we briefly describe the scaling procedure that we use to obtain consistency between within-market covariances estimated by a local model and the same covariances estimated by a global model.

Our starting point is a family of local risk models that provide estimates of the covariances among the factors within each model. Let us denote these factor covariance matrices $F_{11}, F_{22},$ etc. In order to build a global model, we specify a set of global factors that allow us to capture commonalities in the local factor returns across models. In addition to providing estimates of the covariance of local factors across different models (denoted by $\hat{F}_{12}, \hat{F}_{23},$ etc.) the global factor approach also provides estimates of the relationship of local factors within each model (denoted by $\hat{F}_{11}, \hat{F}_{22},$ etc.) Let us denote the estimated factor covariance matrix provided by the global model by $\hat{F}$. This matrix contains matrices $\hat{F}_{11}, \hat{F}_{22},$ etc. along the diagonal blocks, and matrices $\hat{F}_{12}, \hat{F}_{23},$ etc. along the off-diagonal blocks. In order to achieve consistency between the local models and the global model, we desire a new factor covariance matrix $F$ that contains the covariance matrices $F_{11}, F_{22},$ etc. along the diagonal.

We proceed as follows, letting:

$$Diag(A) = \text{diagonal matrix whose diagonal elements are those of the matrix } A$$
$$F_{ii} = \text{the block diagonal matrix whose } i^{th} \text{ diagonal block is the factor covariance matrix for market } i, F_{ii}$$
$$\hat{F}_{ii} = \text{the block diagonal matrix whose } i^{th} \text{ diagonal block is the factor covariance matrix for market } i, \hat{F}_{ii}$$

$$A^{1/2} = \text{the symmetric square root of a symmetric positive semi-definite matrix } A$$
$$C(A) = \text{the correlation matrix of } A, \text{ i.e., } C(A) = Diag(A)^{-1/2} A Diag(A)^{-1/2}$$

The desired covariance matrix $F$ is computed as follows:
Appendix B

European Equity Market

1 Introduction.

Barra’s regional European equity model, EUE2, is significantly different from the single-country equity models. The most important distinction is the existence of the country factors that sum up to zero on a square root of capitalization weighted basis. In order to integrate EUE2 into the Barra Integrated Model framework we need to re-state the model in terms of individual single-country European models. In an additional complication, while EUE2 covers both continental Europe and the UK, we would like to use EUE2 in BIM to represent only the 15 continental Western European countries. We would like to use a separate UK equity model to represent the UK market.

We approach integration of EUE2 in BIM as a several steps process.

2 Compute European single country models from EUE2 factors.

First we need to re-cast a regional, multi-country, EUE2 model into a more familiar single-country mold. For each country $e$ in continental Europe, create a set of “single country” industry and risk factors, $f_{\text{ind}_e,k}$ and $f_{\text{rsk}_e,m}$, from EUE2 factors as follows:

$$
f_{\text{ind}_e,k} = f_{\text{eue2 entry}_e} + f_{\text{eue2 ind}_k} \quad (1)
$$

$$
f_{\text{rsk}_e,m} = f_{\text{eue2 rsk}_m} \quad (2)
$$

In these equations, $f_{\text{eue2 entry}_e}$ is the EUE2 country factor return for country $e$, $f_{\text{eue2 ind}_k}$ is the EUE2 continental industry factor return for industry $k$, and $f_{\text{eue2 rsk}_m}$ is the return of EUE2 risk factor $m$. 

$$
F = \text{Diag}(F_D)^{1/2} C(F_D)^{1/2} C(\hat{F}_D)^{-1/2} C(\hat{F}) C(F_D)^{1/2} C(F_D)^{1/2} \text{Diag}(F_D)^{1/2} 
$$

(1)
Equations (1) and (2) allow us to create a single-country representation of EUE2 model. One would like, however, to be able to go the other way as well so that knowing the single-country representation one could back out the original EUE2 factor returns. Unfortunately, since country and industry factors in Equations (1) and (2) always occur in a sum, the model is not well defined. One can add 1% to each country factor, \( f_{\text{eue2\_cntry}} \), and compensate exactly by subtracting 1% from each industry factor, \( f_{\text{eue2\_ind}} \). To get a unique reverse mapping from the European single-country factors to EUE2 factors we introduce a European market factor, \( f_{\text{eur}} \). Returns to this factor are computed as follows:

\[
f_{\text{eur}} = \frac{\sum_{k=1}^{K} w_k \times f_{\text{eue2\_ind}}}{\sum_{k=1}^{K} w_k}
\]

where

\[
w_k = \sum_{i=1}^{N} \sqrt{\tilde{w}_i} x_{i,k}
\]

and

\( f_{\text{eue2\_ind}} \) = EUE2 factor return for (continental) industry \( k \)

\( N \) = number of EUE2 estimation universe assets

\( K \) = number of EUE2 industries

\( \tilde{w}_i \) = market capitalization of asset \( i \)

\( x_{i,k} \) = exposure of asset \( i \) to industry \( k \).

This factor represents a return to an aggregate portfolio of EUE2 equity. Notice that it follows from the EUE2 structure that \( f_{\text{eur}} \) is the same if calculated over continental or UK industries in EUE2. We calculate it using continental EUE2 industries.
Equations (1), (2), and (3) together define a matrix of exposures of expanded, single-country, set of EUE2 local factors to the original set of EUE2 factors, $H$:

$$ f_e = H f_{eue2} , $$

(5)

where $f_e = (f_{inde} \ f_{rske} \ f_{eur})'$ and $f_{eue2}$ are the EUE2 factor returns.

Note that

$$ f_{eue2} = \left( H^t H \right)^{-1} H^t f_e . $$

(6)

In the following, we will use a notation $P = \left( H^t H \right)^{-1} H^t$.

### 3 Computing global factor returns.

To set up the global equity regression we need the exposures of single-country EUE2 factors to the global BIM factors. For the industry and risk factors the exposure structure is the same as that for the rest of (genuine) single-country models:

$$ f_{ind_{e,k}} = g_w + g_e + g_{ind_k} + \phi_{e,k} $$

(7)

$$ f_{rsk_{e,m}} = g_{rsk_m} + \phi_{e,m} $$

(8)

$g_w$ = return to the World global factor.

$g_e$ = return to individual European global country factors.

$g_{ind_k}$ = return to $k$-th global industry factor.

$g_{rsk_m}$ = return to $m$-th global risk factor.

$\phi_{e,k}, \phi_{e,m}$ = purely local European industry and risk returns in single-country representation

We model $f_{eur}$ as:
\[ f_{\text{eur}} = \sum_{e \in \text{Europe}} w_e g_e + \varphi_{\text{eur}} \]  \hspace{1cm} (9)

where

\[ w_e = \frac{\sum_{k \in e} w_{e,k}}{\sum_{e \in \text{Europe}} w_{e,k}} \]  \hspace{1cm} (10)

\[ w_{e,k} = \sum_{i \in e} \sqrt{w_i} x_{i,k} \]

\( \varphi_{\text{eur}} \) = purely local European market return

We expose \( f_{\text{eur}} \) to UK global country factor as well since by EUE2 construction market return in the continental Europe is affected by the UK market return.

Equations (7), (8), and (9) define BIM global equity regression. Finally, we need to decide on the weighting scheme in the regression:

1. The weight given to each single-country industry factor is given by Equation (4).
2. The weight given to each one of the risk factors is proportional to country capitalization. Since Equation (8) is identical for risk indices in each one of the individual European countries, we simply sum them up and include a single equation for each one of the original EUE2 risk factors in the global regression, weighted by the combined capitalization of all continental Western European countries.
3. The weight for the European market factor \( f_{\text{eur}} \) in the global regression is zero.

4 Computing covariance matrices

As usual, BIM covariance matrix is given by Equation (6). Global regression provides us with the purely local factor return for the single-country factors. However, it is obvious that these returns are not independent since the underlying EUE2 model has far fewer factors than its single-country representation. In effect, these returns are purely local for the combined European market relative to the rest of the world but they are not purely local inside the European market. For example, purely local returns for Austria may be correlated with the returns for France. An assumption of block-diagonal structure of the European sub-matrix of \( \Phi \) is unjustified. Instead, we must convert European purely
local returns back into EUE2 representation using Equation (6). If we define a vector of European single-country factor residuals as \( \phi_e = (\phi_{e,k}, \phi_{e,m}, \phi_{eur})' \) then the conversion is given by \( \phi_{\text{eue2}} = P\phi_e \). We then compute the covariance matrix of purely local returns, \( \Phi_{\text{eue2}} \), in a usual way using EM algorithms and exponential weighting with 48-month half-life.

Since we now have residual covariance matrix in EUE2 space, we need to convert \( YGY' \) into that space as well. Again, we are using the inverse linear transformation matrix \( P \) so that covariance matrix becomes:

\[
F = \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} YGY' \begin{pmatrix} P' & 0 \\ 0 & I \end{pmatrix} + \Phi_{\text{eue2}}.
\]  

(11)

Here, \( \Phi_{\text{eue2}} \) is a block-diagonal covariance matrix of purely local factors where EUE2 block replaced covariance matrices for the individual Western European countries as described above.
Appendix C

Global and Core Factors

Table 1 Equity factors: G – global, C – core, E – through regional European model.

<table>
<thead>
<tr>
<th>World</th>
<th>Country</th>
<th>Industry and Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 World</td>
<td>G C</td>
<td></td>
</tr>
<tr>
<td>2 Australia</td>
<td>G C 30 Poland</td>
<td>2 Automobiles</td>
</tr>
<tr>
<td>3 Bahrain</td>
<td>G C 31 Russia</td>
<td>3 Construction</td>
</tr>
<tr>
<td>4 Brazil</td>
<td>G C 32 South Africa</td>
<td>4 Conglomerate</td>
</tr>
<tr>
<td>5 China</td>
<td>G C 33 Singapore</td>
<td>5 Electricity</td>
</tr>
<tr>
<td>6 Chile</td>
<td>G C 34 Slovakia</td>
<td>6 Energy</td>
</tr>
<tr>
<td>7 Canada</td>
<td>G C 35 Saudi Arabia</td>
<td>7 Financial Services</td>
</tr>
<tr>
<td>8 Colombia</td>
<td>G 36 Thailand</td>
<td>8 Food</td>
</tr>
<tr>
<td>9 Czech Republic</td>
<td>G 37 Sweden</td>
<td>9 Heavy Manufacturing</td>
</tr>
<tr>
<td>10 Denmark</td>
<td>G C 38 Switzerland</td>
<td>10 Insurance</td>
</tr>
<tr>
<td>11 Egypt</td>
<td>G C 39 Turkey</td>
<td>11 Information Technology</td>
</tr>
<tr>
<td>12 Hong Kong</td>
<td>G C 40 Taiwan</td>
<td>12 Light Manufacturing</td>
</tr>
<tr>
<td>13 Hungary</td>
<td>G 41 UK</td>
<td>13 Materials</td>
</tr>
<tr>
<td>14 Indonesia</td>
<td>G C 42 US</td>
<td>14 Media</td>
</tr>
<tr>
<td>15 Israel</td>
<td>G C 43 Venezuela</td>
<td>15 Mining</td>
</tr>
<tr>
<td>16 India</td>
<td>G C 44 Zimbabwe</td>
<td>16 Pharmaceuticals and Health</td>
</tr>
<tr>
<td>17 Jordan</td>
<td>G 45 Austria</td>
<td>17 Precious Metals</td>
</tr>
<tr>
<td>18 Japan</td>
<td>G C 46 Belgium</td>
<td>18 Property</td>
</tr>
<tr>
<td>19 Korea</td>
<td>G C 47 Finland</td>
<td>19 Telecommunications</td>
</tr>
<tr>
<td>20 Sri Lanka</td>
<td>G 48 France</td>
<td>20 Transportation</td>
</tr>
<tr>
<td>21 Malaysia</td>
<td>G C 49 Germany</td>
<td>21 Travel and Entertainment</td>
</tr>
<tr>
<td>22 Morocco</td>
<td>G C 50 Ireland</td>
<td>22 Textiles</td>
</tr>
<tr>
<td>23 Mexico</td>
<td>G C 51 Italy</td>
<td>23 Utilities</td>
</tr>
<tr>
<td>24 Nigeria</td>
<td>G C 52 Netherlands</td>
<td>1 Size</td>
</tr>
<tr>
<td>25 New Zealand</td>
<td>G C 53 Portugal</td>
<td>2 Momentum</td>
</tr>
<tr>
<td>26 Oman</td>
<td>G C 54 Spain</td>
<td>3 Volatility</td>
</tr>
<tr>
<td>27 Peru</td>
<td>G C 55 Greece</td>
<td></td>
</tr>
<tr>
<td>28 Philippines</td>
<td>G C 56 Norway</td>
<td>4 Value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Currency factors: G – global, C – core, M€ - part of European currency model, € - pegged to Euro, $ - pegged to US Dollar, N – model numeraire.

<table>
<thead>
<tr>
<th>Currency</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Euro</td>
<td>36 Lithuanian Lita</td>
</tr>
<tr>
<td>2 Argentinean Peso</td>
<td>37 Malaysian Ringgit</td>
</tr>
<tr>
<td>3 Austrian Schilling</td>
<td>38 Maltese Lira</td>
</tr>
<tr>
<td>4 Australian Dollar</td>
<td>39 Moroccan Dirham</td>
</tr>
<tr>
<td>5 Belgian Franc</td>
<td>40 Mexican Peso</td>
</tr>
<tr>
<td>6 Bahraini Dinar</td>
<td>41 Dutch Guilder</td>
</tr>
<tr>
<td>7 Brazilian Real</td>
<td>42 Nigerian Naira</td>
</tr>
<tr>
<td>8 Bulgarian Lev</td>
<td>43 Norwegian Krone</td>
</tr>
<tr>
<td>9 Chinese Yuan</td>
<td>44 New Zealand Dollar</td>
</tr>
<tr>
<td>10 Chilean Peso</td>
<td>45 Omani Rial</td>
</tr>
<tr>
<td>11 Canadian Dollar</td>
<td>46 Peruvian Sol</td>
</tr>
<tr>
<td>12 Colombian Peso</td>
<td>47 Philippines Peso</td>
</tr>
<tr>
<td>13 Croatian Kuna</td>
<td>48 Pakistani Rupee</td>
</tr>
<tr>
<td>14 Cyprus Pound</td>
<td>49 Polish Zloty</td>
</tr>
<tr>
<td>15 Czech Koruna</td>
<td>50 Portuguese Escudo</td>
</tr>
<tr>
<td>16 Danish Krone</td>
<td>51 Russian Ruble</td>
</tr>
<tr>
<td>17 Egyptian Pound</td>
<td>52 South African Rand</td>
</tr>
<tr>
<td>18 Spanish Peseta</td>
<td>53 Singapore Dollar</td>
</tr>
<tr>
<td>19 Finnish Markka</td>
<td>54 Slovenian Tolar</td>
</tr>
<tr>
<td>20 French Franc</td>
<td>55 Slovak Koruna</td>
</tr>
<tr>
<td>21 Greek Drachma</td>
<td>56 Swedish Krona</td>
</tr>
<tr>
<td>22 Deutsche Mark</td>
<td>57 Swiss Franc</td>
</tr>
<tr>
<td>23 Estonian Kroon</td>
<td>58 Saudi Arabian Riyal</td>
</tr>
<tr>
<td>24 Hong Kong Dollar</td>
<td>59 Thailand Baht</td>
</tr>
<tr>
<td>25 Hungarian Forint</td>
<td>60 Turkish Lira</td>
</tr>
<tr>
<td>26 Indonesian Rupiah</td>
<td>61 New Taiwan Dollar</td>
</tr>
<tr>
<td>27 Irish Pound</td>
<td>62 British Pound Sterling</td>
</tr>
<tr>
<td>28 Israeli Shekel</td>
<td>63 US Dollar</td>
</tr>
<tr>
<td>29 Indian Rupee</td>
<td>64 Venezuelian Bolivar</td>
</tr>
<tr>
<td>30 Italian Lira</td>
<td>65 Zimbabwean Dollar</td>
</tr>
<tr>
<td>31 Jordan Dinar</td>
<td>66 Lebanese Pound</td>
</tr>
<tr>
<td>32 Japanese Yen</td>
<td>67 Qatari Riyal</td>
</tr>
<tr>
<td>33 Korean Won</td>
<td>68 Ecuadorian Sucre</td>
</tr>
<tr>
<td>34 Sri Lankan Rupee</td>
<td>69 Netherlands Antilles Guilder</td>
</tr>
<tr>
<td>35 Latvian Lat</td>
<td>M€</td>
</tr>
</tbody>
</table>

Table 3 Commodity factors: G – global, C – core

<table>
<thead>
<tr>
<th>Commodity Sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GSCI Agriculture</td>
<td>G C</td>
</tr>
<tr>
<td>2 GSCI Energy</td>
<td>G C</td>
</tr>
<tr>
<td>3 GSCI Industrial Metals</td>
<td>G C</td>
</tr>
<tr>
<td>4 GSCI Livestock</td>
<td>G C</td>
</tr>
<tr>
<td>5 GSCI Precious Metals</td>
<td>G C</td>
</tr>
</tbody>
</table>

© MSCIBarra, 2005
<table>
<thead>
<tr>
<th>Market</th>
<th>Sovereign Term Structure: Shift</th>
<th>Sovereign Term Structure: Twist</th>
<th>Swap Spread: Shift</th>
<th>Inflation-Protected Term Structure: Shift</th>
<th>Average Credit Spread</th>
<th>Implied Volatility</th>
<th>Muni Term Structure: Shift</th>
<th>Muni Term Structure: Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>Denmark</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emerging Markets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euro</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>Japan</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>New Zealand</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Singapore</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>United States</td>
<td>G</td>
<td>C</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hong Kong</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Malaysia</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovakia</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Philippines</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>