Shortfall in Portfolio Construction | June 2009

MSCI Barra RESEARCH BULLETIN

Introduction

Expected shortfall—the average loss beyond a threshold that distinguishes an everyday loss from an extreme loss—is the most direct measure of extreme risk. The threshold is known as the Value at Risk (VaR) and is specified by a confidence level. The 99% confidence level one-day VaR provides an upper bound for portfolio loss for 99 days out of 100. It is the best case scenario for the worst day out of 100, and thus it provides no indication of what to expect in times of severe distress.

On the other hand, the 99% one-day shortfall is an average, or expected value, for the loss on the worst day out of 100. It is a natural guide to potential losses in extreme market conditions. When estimated from a distribution that reflects salient characteristics of empirical data, including a realistic proportion of extreme events and the asymmetry between gains and losses, shortfall provides information that complements volatility, which is a measure of average return dispersion.

Comparison between Normal and Barra Extreme Risk (BXR) Measures of Tail Risk

There are several ways to estimate the expected shortfall of a portfolio. A simple way is to assume that portfolio returns are normally distributed with mean zero and variance σ^2. In this case portfolio shortfall is a multiple of volatility, where the multiplier is a constant that depends on the confidence level. There is, however, a large body of evidence that asset returns are non-normally distributed (see the recent MSCI Barra Research Bulletin *Understanding the Tails of the Return Distribution* for an overview of some stylized facts). Therefore, this approach may not reflect the characteristics of empirical data and, hence, may misestimate extreme losses.

As an alternative, the Barra Extreme Risk (BXR) methodology forecasts expected shortfall based on a long synthetic history of Barra factor returns, reflecting the properties of empirical financial returns, such as asymmetry between gains and losses, heightened correlations during times of turbulence, and a significant proportion of large returns (or losses). Called Barra Extreme Shortfall (xShortfall), this expected shortfall can be expressed as volatility times a multiplier that is a function of portfolio factor exposures, factor return history, and portfolio specific risk.

Figure 1 illustrates the difference between xShortfall and expected shortfall based on the normal distribution between June 2007 and November 2008 for a momentum-tilted European equity portfolio. This portfolio has significant exposure to the momentum factor from the Barra Europe Equity Model (EUE2) , while exposures to industry and other style factors are kept close to benchmark (MSCI Europe Index) levels. We see that the difference between the two shortfall measures first spikes at the beginning of the "quant crisis" in August 2007, then during the sell-off in January 2008, and again during the meltdown of October 2008. This illustrates that xShortfall is more responsive to increased volatility than shortfall based on the normal distribution.

Figure 2 shows that while the difference between normal shortfall and Barra Extreme Shortfall can often be explained by portfolio volatility, this relationship becomes weaker during times of extreme market stress. This can be explained by noting that[1]

$$ES_p^{normal} - ES_p^{Barra} = \sigma\left[\Theta(p)_{normal} - \Theta(p)_{Barra}\right],$$

[1] For more details see the Appendix.

where $\Theta(p)_{normal}$ is a constant and $\Theta(p)_{Barra}$ is close to a constant for many portfolios most of the time. Hence, most changes in the right-hand side of the equation will come as a result of fluctuations in volatility.

Figure 1: Difference between xShortfall and Normal Expected Shortfall

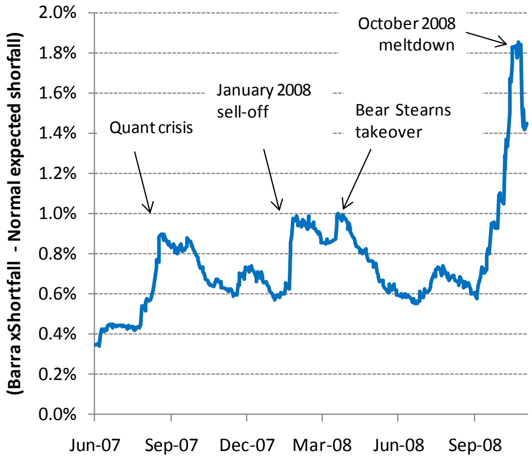
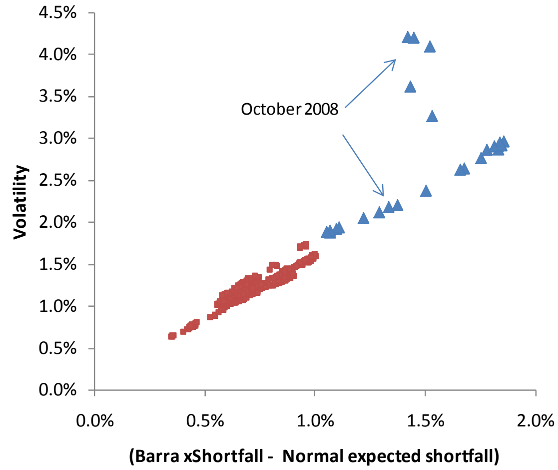


Figure 2: Relationship between xShortfall, Normal Expected Shortfall, and Portfolio Volatility



Examining the Contributions to Shortfall and Volatility

The BXR methodology can offer insight on how the factor exposures of the portfolio contribute to extreme losses. At the factor level, we can examine two sets of statistics: contributions to risk and factor betas. The contribution to risk is a descriptive statistic—it tells the investor, given the current factor exposures, where a loss is likely to come from. We can compare the contributions to volatility and to xShortfall to get an indication of which factor exposures will lead to normal and extreme losses.

To illustrate how portfolio exposures can contribute to normal and extreme losses, we use examples of four different European equity portfolios:

- An index tracking portfolio, which tracks the MSCI Europe Index using only 100 stocks
- A financial industry-tilted portfolio without constraints on style exposures
- A financial industry-tilted portfolio with neutralized style exposures
- A momentum-titled portfolio

We perform the analysis at 95% confidence interval for the beginning of August 2007 and October 2008 (corresponding to the beginning of the “quant crisis” and the recent meltdown, respectively). Tables 1 and 2 illustrate the active contributions to one-day shortfall and volatility by groups of factors (with the MSCI Europe Index as the benchmark).

We can observe from the tables that:

- The largest contributions to active xShortfall and tracking error, on the level of factor groups, often mirror each other.
- Nevertheless, there are occasional exceptions. For example, the contribution of specific risk to xShortfall is usually higher than the contribution of specific risk to tracking error. In August

2007, specific risk had the biggest contribution to the active xShortfall for the index tracking portfolio, while style risk had the biggest contribution to tracking error.

- Typically, for the index tracking portfolio the biggest contributions to both xShortfall and tracking error come from risk indices and specific risk; for the industry-tilted portfolios the biggest contributions come from industry factors; and for the momentum-tilted portfolio the biggest contributions come from risk indices and specific risk.
- The relative importance of factor groups can vary through time. For example, risk index exposure becomes relatively less important for the index tracking portfolio in October 2008 compared to August 2007.
- Interestingly, in October 2008 country factor contributions to tracking error and to xShortfall often differ in sign and magnitude.

Table 1: Active Contributions, August 2007

	Index tracking		Industry tilt		Industry tilt (style neutral)		Momentum	
	xShortfall	Tracking error	xShortfall	Tracking error	xShortfall	Tracking error	xShortfall	Tracking error
Risk indices (%)	52.69%	66.57%	8.09%	13.79%	-5.52%	1.66%	21.70%	42.33%
Industries (%)	-12.10%	-6.49%	61.54%	64.97%	67.91%	72.69%	20.80%	16.90%
Countries (%)	-1.28%	0.61%	-3.71%	-0.95%	-0.67%	1.30%	7.17%	6.57%
Currencies (%)	-2.70%	-1.45%	2.15%	2.43%	1.59%	1.78%	-0.77%	-0.14%
Specific (%)	63.40%	40.76%	31.93%	19.76%	36.69%	22.56%	51.10%	34.33%
Forecast active xShortfall	0.12%		0.31%		0.32%		0.43%	
Forecast TE	0.06%		0.15%		0.15%		0.21%	

Table 2: Active Contributions, October 2008

	Index tracking		Industry tilt		Industry tilt (style neutral)		Momentum	
	xShortfall	Tracking error	xShortfall	Tracking error	xShortfall	Tracking error	xShortfall	Tracking error
Risk indices (%)	30.46%	30.79%	-4.30%	-1.12%	-5.39%	-3.44%	52.61%	52.93%
Industries (%)	3.03%	7.29%	67.97%	74.62%	62.78%	71.28%	2.67%	8.60%
Countries (%)	-9.73%	0.44%	-1.91%	1.47%	0.54%	4.31%	-2.91%	0.67%
Currencies (%)	-8.88%	-2.45%	-0.48%	0.31%	-2.19%	-0.50%	-0.92%	-0.65%
Specific (%)	85.11%	63.94%	38.72%	24.73%	44.27%	28.34%	48.56%	38.45%
Forecast active xShortfall	0.24%		0.79%		0.87%		0.92%	
Forecast TE	0.12%		0.37%		0.41%		0.43%	

Managing Shortfall and Volatility

Factor betas allow us to determine the least and most diversifying factor positions with respect to both volatility and xShortfall. Factor beta can be expressed as the marginal contribution to risk as a fraction of total risk.[2] The betas describe what happens to a factor on a bad day (xShortfall beta) or an average day (volatility beta). This is an actionable statistic—the marginal contribution describes the impact of making small modifications to exposures.

As an illustration, Figure 3 plots in total space the style factor exposures, as well as the style factor volatility and xShortfall betas, for the momentum-tilted portfolio as of August 1, 2007. As expected, the largest style exposure is to the momentum factor, while the other exposures are at modest levels. We observe that for the variability factor the xShortfall beta is above the volatility beta, meaning that reducing variability exposure would have a larger impact on xShortfall relative to volatility.

Table 3 illustrates what happens if we rebalance the portfolio to reduce the variability exposure while keeping the other style exposures unchanged.[3] This reduces both the forecast volatility and the forecast xShortfall of the portfolio; however, as expected, the change in xShortfall is more pronounced. We also see that the realized maximum loss for the rebalanced portfolio during the subsequent turbulent month of August 2007 was considerably lower.

Figure 3: Style Factor Exposures and Betas, Momentum-Tilted Portfolio, 1 August 2007

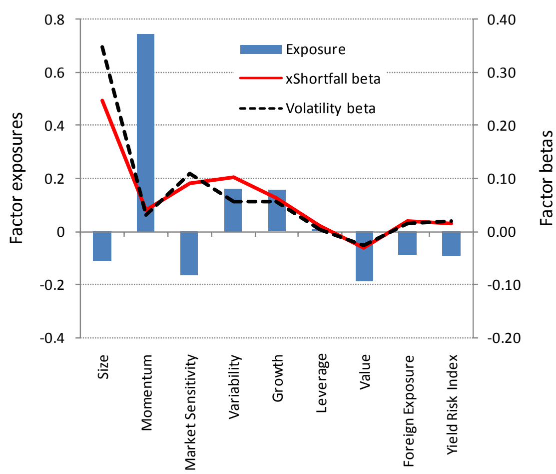


Table 3: Effect of Modifying Portfolio Exposure to Variability Factor

	Momentum portfolio	Modified portfolio
Variability exposure	0.16	-0.34
Forecast risk	0.92%	0.85%
Forecast xShortfall	2.46%	2.21%
Realized std. dev (August 2007)	1.71%	1.27%
Maximum loss (August 2007)	4.22%	3.35%

Conclusion

In this bulletin we have explored the meaning of the statistics produced by the Barra Extreme Risk (BXR) methodology and illustrated how Barra Extreme Shortfall (xShortfall), forecast from a factor model, can provide useful information that complements volatility. Forecasting extreme risk through a multi-factor framework not only provides the advantage of more accurately reflecting the return properties of the portfolio based on up-to-date exposures, but also gives helpful information regarding portfolio exposures during extreme market turmoil.

2 See the Appendix for more details about the calculation of the marginal contribution to xShortfall.
3 Note that this also resulted in a significant change in the exposure to the Continental Banking industry factor. However, the volatility and xShortfall betas are similar for this factor.

Appendix

Comparison between Normal and Barra Extreme Risk (BXR) Measures of Shortfall

Formally, we can express portfolio shortfall as

$$ES_P = E\left[L \mid L > VaR_p\right],$$

where $L = -(R - E[R])$ is the excess portfolio loss and VaR_p is portfolio Value at Risk. If we assume that portfolio returns are normally distributed, then the Value at Risk with confidence level p is given by

$$VaR_p^{normal} = \sigma\Phi^{-1}(p)$$

and expected shortfall can be calculated as

$$ES_p^{normal} = \sigma\Theta(p)_{normal}.$$

$\Theta(\bullet)_{normal}$ is the normal multiplier, defined as

$$\Theta(p)_{normal} = \frac{\phi\left[\Phi^{-1}(p)\right]}{p},$$

where $\phi(\bullet)$ denotes the density function and $\Phi(\bullet)$ the cumulative density function of the standard normal distribution.

Barra Extreme Shortfall (xShortfall) can be expressed as

$$ES_p^{Barra} = \sigma\Theta(p, X, f, \Delta)_{Barra},$$

where $\Theta(\bullet)_{Barra}$ is the BXR multiplier, estimated using the rescaled empirical history of factor returns.

Using portfolio exposures, factor returns, and portfolio specific risk, the BXR methodology generates a synthetic history of portfolio returns. These returns are normalized to remove volatility dynamics and rescaled by the current volatility level to create a series that is independently identically distributed (iid). From the rescaled historical returns, a distribution is forecast using the peaks over thresholds method, which takes the core of the distribution to be an empirical histogram and fits the tails to a generalized Pareto distribution (for details see Goldberg, Miller, and Weinstein, 2008).

Portfolio Analytics in the BXR Methodology

A risk factor's marginal contribution to shortfall is the partial derivative of portfolio shortfall with respect to the risk factor exposure x_i

$$MCR_i^{shortfall} = \frac{\partial ES_p}{\partial x_i}.$$

As shown by Goldberg et al. (2009), it can be expressed as

$$\frac{\partial ES_p}{\partial x_i} = E\left[l_i \mid L > VaR_p\right],$$

where l_i is the excess loss of factor i. Empirically, therefore, the marginal contribution can be calculated as the average loss of a factor on days when the portfolio Value at Risk is breached.

The contribution to shortfall risk for factor i is defined as

$$RC_i^{shortfall} = \frac{x_i \times MCR_i^{shortfall}}{\sum_i x_i \times MCR_i^{shortfall}}$$

where x_i is the portfolio factor exposure and $MCR_i^{shortfall}$ is the marginal contribution to shortfall risk of factor i. The contribution to volatility can be defined in a similar manner.

Shortfall betas are defined as

$$\beta_i = \frac{MCR_i^{shortfall}}{ES_p}.$$

We can define volatility betas similarly and compare the values of shortfall and volatility betas directly, as the marginal contributions are rescaled by total risk. Beta gives the fractional change in risk for a given change in exposure to the factor. For more details, please see Menchero and Poduri (2008) and Goldberg et al. (2009).

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