130-30 Implementation Challenges

Understanding the Performance Drivers of a Short Extension Strategy

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Introduction

Short extension strategies, also known as active extension strategies or 130/30 strategies, bridge the gap between traditional long-only portfolios and unconstrained long-short hedge funds by allowing limited leverage and shorting to be used in the construction of the portfolio. In these strategies, the overall portfolio remains fully exposed to the equity market, in the sense that the net market exposure across the portfolio remains 100%.

In this paper we examine some important parameters affecting the implementation of short extension strategies. First, we analyse the relationship between portfolio leverage and active risk and show that there is an optimal level of leverage for a given level of active risk. Then, we show how changes in the investment universe or in market conditions could affect the performance of short extension portfolios. Finally, we explore the question of how different investment processes could be affected by short extension implementation.

Defining Leverage

Given a portfolio with active weights h_A relative to benchmark weights h_B we define leverage as the excess amount invested in long and short positions per unit of capital:

$$L = \underbrace{\sum_{i=1}^{N} |\underbrace{\mathrm{h}_{A,i} + \mathrm{h}_{B,i}}_{\text{weight of asset i}}|}_{\substack{\text{Total amount invested in} \\ \text{long and short positions}}} - 1$$

In a long-only portfolio where all weights are positive we can show that leverage is zero:

$$L = \sum_{i=1}^{N} \left| h_{A,i} + h_{B,i} \right| - 1 = \sum_{i=1}^{N} \left(h_{A,i} + h_{B,i} \right) - 1 = \underbrace{\sum_{i=1}^{N} h_{A,i}}_{=0} + \underbrace{\sum_{i=1}^{N} h_{B,i}}_{=1} - 1 = 0$$

In a long-short portfolio where some weights are positive and some are negative, leverage will be strictly positive:

$$L = \sum_{\substack{\text{asset } j \\ \text{is short}}} \left| h_{A,j} + h_{B,j} \right| + \sum_{\substack{\text{asset } i \\ \text{is long}}} \left| h_{A,i} + h_{B,i} \right| - 1 = \overbrace{\underbrace{\sum_{i=1}^{N} \left(h_{A,i} + h_{B,i} \right)}_{=1} - 1}^{=0} - 2 \underbrace{\sum_{\substack{\text{asset } j \\ \text{is short}}} \left(h_{A,j} + h_{B,j} \right)}_{<0} > 0$$

Following this last derivation, we obtain an equivalent expression for leverage:

$$L = -2 \sum_{\substack{\text{asset j} \\ \text{is short}}} \left(h_{A,j} + h_{B,j} \right)$$

In the particular case of a 130/30 portfolio our leverage definition implies 60% leverage:

$$L = \underbrace{\sum_{\substack{\text{asset } j \\ \text{is short}}} \left| h_{A,j} + h_{B,j} \right|}_{30\%} + \underbrace{\sum_{\substack{\text{asset } i \\ \text{is long}}} \left| h_{A,i} + h_{B,i} \right|}_{130\%} - 1 = 60\%$$

Optimal Leverage

One of the main challenges in short extension portfolio construction is to determine the appropriate level of leverage. For example, is 130/30 more appropriate than 120/20 or 140/40? What parameters should guide the choice of leverage? In order to gain insights into this question we start by considering a simple two-asset world and solve the following unconstrained long-short optimization problem:

$$\max_{h_A}\left\{h_A{}'\alpha-\frac{1}{2}\lambda h_A{}'Vh_A\right\}$$

$$Subject\ to: h_{A,1}+h_{A,2}=0$$

Here, $\alpha=(\alpha_1,\alpha_2)$ is the vector of active returns, $h_A=(h_{A,1},h_{A,2})$ is the vector of active weights, λ is a risk-aversion parameter, and *V* is the covariance matrix of asset returns. Without loss of generality we can assume that $\alpha_1>\alpha_2$. Under this assumption, the active weights of the optimal long-short portfolio are given by the following expression:

$$h_{A1}=-h_{A2}=\frac{\alpha_1-\alpha_2}{\lambda(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)}=\frac{\sigma_A}{(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^{1/2}} \qquad (1)$$

Here, σ_1 and σ_2 are the volatilities of the two assets, ρ is the correlation between the two assets and σ_A is the active risk of the portfolio, which is given by the following equation:

$$\sigma_A=\left(h_A{}'Vh_A\right)^{1/2}=\frac{\alpha_1-\alpha_2}{\lambda(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^{1/2}}$$

Equation (1) highlights the direct relationship between active risk and optimal active weights and reveals that the asset with higher alpha, in this case asset 1, has positive active weight while the lower alpha asset, in this case asset 2, has negative active weight. The portfolio weight of asset 2 can be either positive or negative. More specifically, the portfolio weight of asset 2 will be positive (negative) if its benchmark weight is higher (lower) than the absolute value of its optimal active weight. This reasoning leads us to express the leverage of the optimal long-short portfolio as follows:

$$L^*=\frac{\sigma_A}{(\sigma_1{}^2+\sigma_2{}^2-2\rho\sigma_1\sigma_2)^{1/2}}(\varepsilon_1-\varepsilon_2)+\varepsilon_1 h_{B,1}+\varepsilon_2 h_{B,2}-1$$

where

$$\varepsilon_i=1 \quad if\ \ asset\ i\ is\ long, in\ other\ words\ if\ \ h_{A,i}+h_{B,i}\geq 0$$

$$\varepsilon_i=-1\,if\ \ asset\ i\ is\ short, in\ other\ words\ if\ \ h_{A,i}+h_{B,i}<0$$

The above expression unveils the one-to-one mapping between leverage and active risk and shows that leverage is increasing with active risk. Indeed, we can rewrite leverage as:

$$L^* = \begin{cases} 0 & \text{for } \sigma_A < \sigma^* \\ \dfrac{2\sigma_A}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)^{1/2}} + h_{B,1} - h_{B,2} - 1 & \text{for } \sigma_A \geq \sigma^* \end{cases}$$

where $\sigma^* = h_{B,2}(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)^{1/2}$

Regardless of benchmark weights, asset 2 is likely to be held long $(\varepsilon_2 = 1)$ for low levels of active risks ($\sigma_A < \sigma^*$) in which case there is no need to resort to leverage as the optimal portfolio is long-only. On the other hand, as active risk increases to higher levels ($\sigma_A > \sigma^*$), the optimal portfolio has a short position in asset 2 ($\varepsilon_2 = -1$). As a consequence, leverage also increases and eventually becomes piece-wise linear[1] in active risk.

Imposing Constraints on Leverage

In this section we examine the implications of imposing constraints on leverage and active risk. We start by considering the same two-stock portfolio optimization problem but with an additional constraint on the level of active risk:

$$\max_{h_A} \left\{ h_A{}'\alpha - \frac{1}{2}\lambda h_A{}'Vh_A \right\}$$

$$\text{Subject to: } \quad (i)\; h_{A,1} + h_{A,2} = 0$$

$$(ii)\; h_A{}'Vh_A = \sigma_A^2$$

The resulting active weights and corresponding optimal leverage remain unchanged compared to the previous unconstrained case when expressed as a function of active risk. In particular, assuming that we are targeting active risk levels higher than the threshold σ^*, the associated optimal level of leverage is:

$$L^* = \frac{2\sigma_A}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)^{1/2}} + 2(h_{B,1} - 1)$$

Setting leverage above the required level L^* for the given target active risk σ_A will lead to sub-optimal portfolios and lower information ratios. Likewise, setting leverage below L^* will also lead to lower information ratios. Indeed, as shown in the appendix, fixing leverage at a different level than the optimal leverage L^* associated with the active risk target can be seen as placing an additional constraint in the portfolio optimization problem which in turn lowers the information ratio.

[1] As shown in the appendix, in general, leverage is convex, piece-wise linear and increasing in active risk and becomes linear when active risk reaches a threshold above which all assets with negative active weights are held short in the optimal portfolio.

Now we remove the active risk constraint and replace it with a fixed leverage constraint:

$$\max_{h_A}\left\{h_A'\alpha-\frac{1}{2}\lambda h_A'Vh_A\right\}$$

$$Subject\ to:\quad (i)\ h_{A,1}+h_{A,2}=0$$

$$(ii)\ \left|h_{A,1}+h_{B,1}\right|+\left|h_{A,2}+h_{B,2}\right|-1=L$$

The resulting active weights are then given by:

$$h_{A,1}=-h_{A,2}=\frac{L}{2}+1-h_{B,1}$$

This in turn implies a corresponding optimal level of active risk:

$$\sigma_A*=(\sigma_1^{\ 2}+\sigma_2^{\ 2}-2\rho\sigma_1\sigma_2)^{1/2}(\frac{L}{2}+1-h_{B,1})$$

Targeting higher levels or lower levels of active risk compared to the optimal level σ_A^* will lead to suboptimal portfolios and lower information ratios.

Table 1: Optimal Portfolio IR for Various Leverage and Tracking Error Constraints

Investment Strategy	Leverage Constraint	Tracking Error Constraint					
		0.5%	1.0%	1.5%	2.0%	4.0%	8.0%
Long-Only	0%	0.82	0.75	0.69	0.64	0.53	0.41
110/10	20%	0.87	0.87	0.82	0.77	0.62	0.48
120/20	40%	0.79	0.88	0.86	0.83	0.69	0.53
130/30	60%	0.71	0.85	0.88	0.86	0.73	0.57
150/50	100%	0.61	0.61	0.86	0.88	0.80	0.64
Long-Short	No Limit	0.89	0.89	0.89	0.89	0.89	0.89

Chart 1

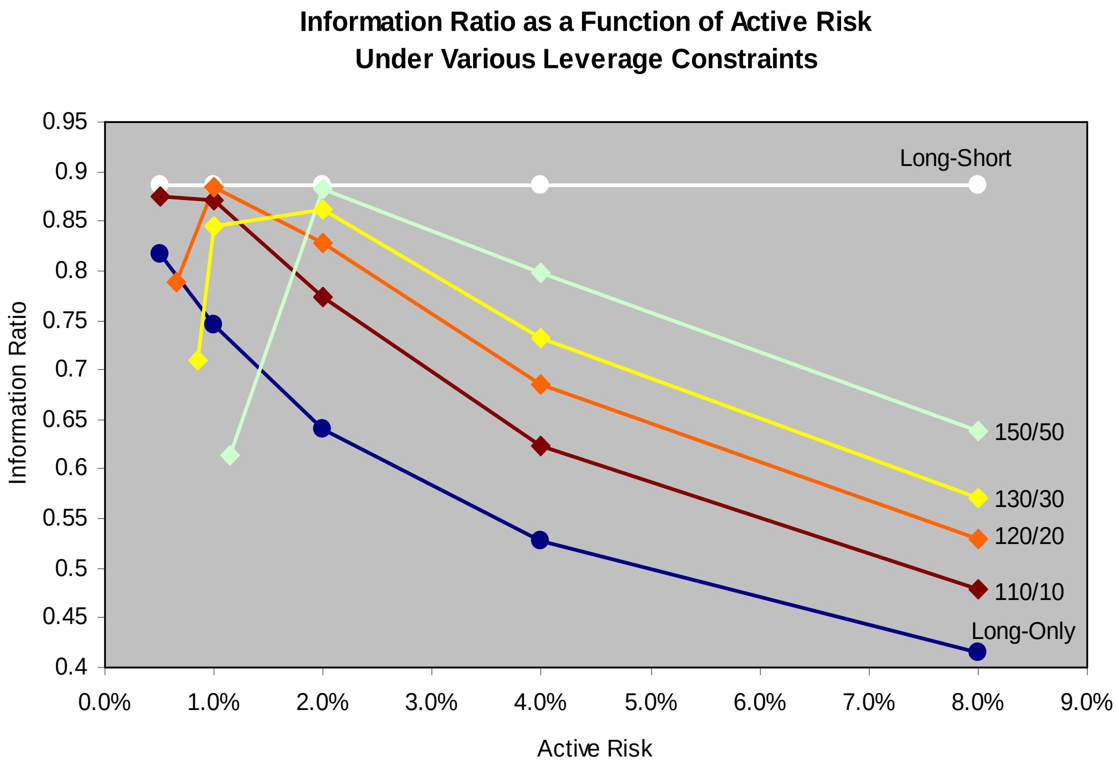


Table 1 and Chart 1 illustrate the relationship between portfolio leverage and active risk and show that there is an optimal level of leverage for a given level of active risk. In this example, we used the latest Barra European Equity model (EUE2L) and the Barra Aegis application to construct optimal long-short portfolios with 100% net exposure to the equity market. The stock selection universe for these portfolios comprised the constituents of the MSCI Europe Index as of April 30, 2007 (approximately 600 stocks) and we derived alphas based on stock exposures to the risk model's Value Risk Index.

In the example shown in Table 1 and Chart 1, the maximum information ratio that can be achieved through unconstrained long-short optimization is 0.89. In the absence of any leverage or other constraints, this maximum information ratio remains constant across all levels of active risk (see last row in the table). The same maximum information ratio can also be achieved with a 120/20 short extension portfolio that has active risk of 1.0%. In other words, 40% is the optimal level of leverage for this particular level of active risk. If the manager wishes to increase the active risk of the portfolio, for example from 1.0% to 1.5%, the optimal level of leverage for this higher level of active risk is 60%. Imposing higher or lower leverage at this level of active risk would lead to a decline in the information ratio of the portfolio.

Impact of Different Investment Universes and Changing Market Conditions

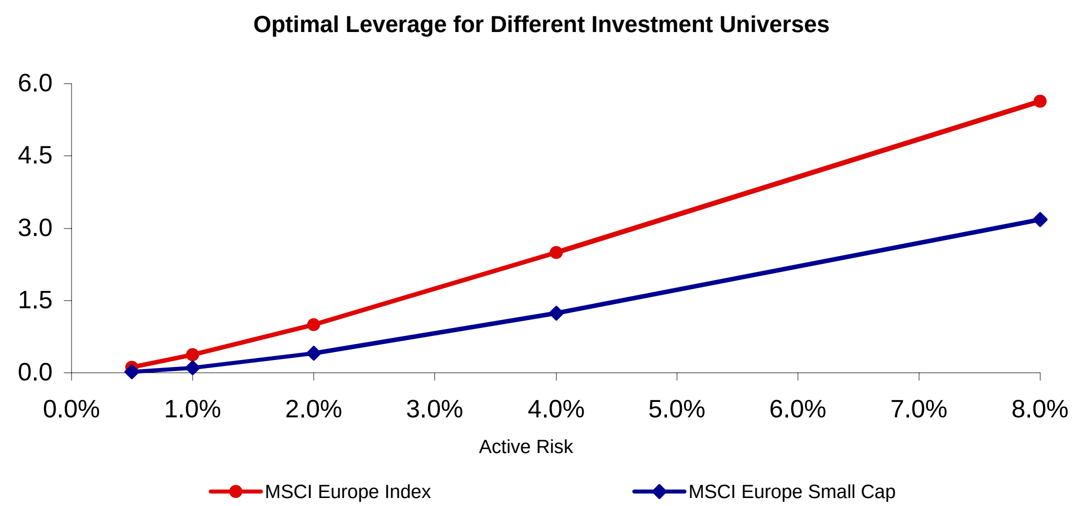
Changing market conditions and different investment universes will generally imply different levels of volatility for the constituents of the portfolio. In turn, this parameter affects the optimal level of leverage required in a long-short portfolio for a given level of active risk. In particular, lower volatilities lead to higher optimal leverage for a given level of active risk. Higher required leverage for a given level of active risk implies higher gain in information ratio from relaxing the long-only constraint.

For example, large cap and developed market portfolios may require a higher level of leverage and therefore may benefit more from short extension implementation, compared to small cap and emerging market portfolios[2], at each given level of active risk. For the same reason, lower leverage would be required for a given level of active risk during periods of extreme events, such as September 2001, or periods of financial market crisis, such as August 2007, when all assets tend to experience higher volatility.

Table 2: Optimal Long-Only & Long-Short Portfolio IR Based on Different Indices

	MSCI Europe Standard Index			MSCI Europe Small Cap Index		
Active Risk	**Long-only IR**	**Long-short IR**	**% Gain in IR**	**Long-only IR**	**Long-short IR**	**% Gain in IR**
0.5%	0.82	0.89	9%	1.11	1.12	1%
1.0%	0.75	0.89	19%	1.08	1.12	4%
2.0%	0.64	0.89	39%	1.01	1.12	12%
4.0%	0.53	0.89	68%	0.88	1.12	27%
8.0%	0.41	0.89	117%	0.69	1.12	63%

Chart 2

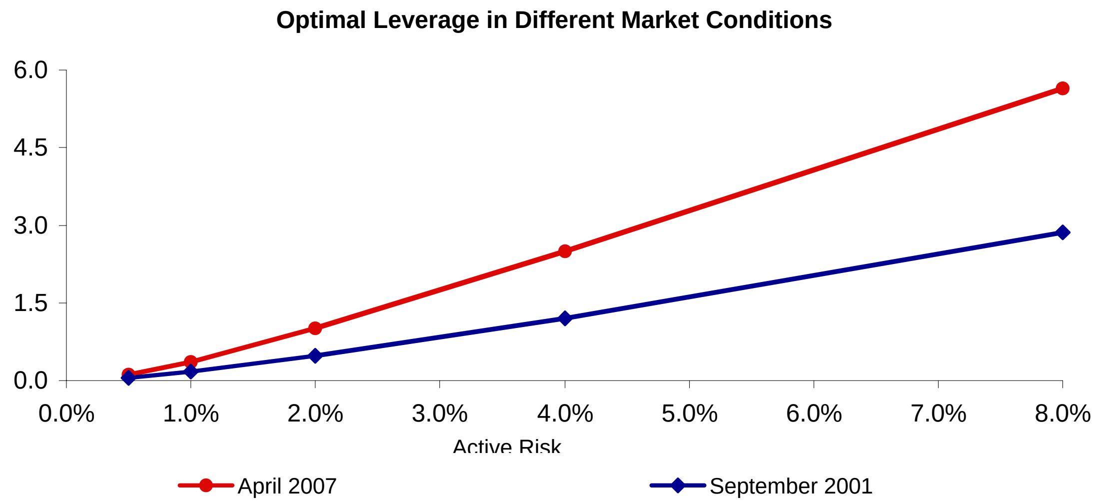


2 These conclusions hold under the assumption of investment universes that have a similar number of stocks and alpha processes for each universe that yield a similar alpha distribution.

Table 2 and Chart 2 present an example that illustrates the relationship between the underlying universe and the potential gain in information ratio from relaxing the long-only constraint. In this example, we used the latest Barra European Equity model (EUE2L) and the Barra Aegis application to construct optimal long-short portfolios with 100% net exposure to the equity market. We report optimal leverage and gain in information ratio for two different stock selection universes, the MSCI Europe Index and the MSCI Europe Small cap Index. Both indices have approximately 600 stocks and in this example we used data as of April 30, 2007.

These results suggest that optimal large cap portfolios may require higher leverage and may enjoy higher percentage gain in information ratio compared to small cap portfolios for the same level of active risk. Chart 3 illustrates the same point by comparing the level of optimal leverage for unconstrained long-short portfolios based on the MSCI Europe Index and constructed at the end of September 2001 (extremely volatile market conditions) and April 2007 (normal market conditions).

Chart 3



The common ingredient driving the results of these case studies is that, everything else being equal, lower leverage is required for investment universes or market conditions characterized by higher levels of stock volatility. Indeed, Table 3 shows the shift in the distribution of stock volatility when going from large cap to small cap stock universes in both the US and Europe, and for the MSCI Europe Index constituents, across the different market environments at the end of April 2007 (normal market conditions) and September 2001 (extreme market conditions).

Table 3: Cross Sectional Distribution of Stock Risk in Different Stock Universes and Market Conditions

	S&P600 Small Cap	S&P500	MSCI Europe Small Cap	MSCI Europe	September 2001	April 2007
10th Percentile	21.6	16.2	24.0	17.8	28.0	17.8
25th Percentile	26.9	18.4	27.2	20.6	31.4	20.6
50th Percentile	32.4	22.8	32.0	23.8	36.2	23.8
75th Percentile	38.5	27.6	37.7	28.3	45.1	28.3
90th Percentile	45.1	33.4	46.4	33.2	56.3	33.2

In order to see why these results are not limited to these particular examples but hold more generally, we recall that in our two-asset framework[3] optimal leverage is expressed as:

$$L^* = \frac{2\sigma_A}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)^{1/2}} + 2(h_{B,1} - 1)$$

Now, consider two environments *E* and *E'* such that the distribution of volatilities in *E'* is shifted rightwards as follows:

$$\begin{cases} \sigma_1' = k\sigma_1 \\ \sigma_2' = k\sigma_2 \end{cases} \quad with\ k > 1$$

where (σ_1^2, σ_2^2) and $(\sigma_1'^2, \sigma_2'^2)$ are the asset return volatilities in environments *E* and *E'*. Given similar benchmark weight distributions and similar asset return correlations, the difference in leverage in the two environments relative to active risk can be expressed as:

$$\frac{L^{*\prime} - L^*}{\sigma_A} = \underbrace{(\frac{1}{k} - 1)}_{<0} \frac{2}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)^{1/2}} < 0$$

In other words, environment *E'* requires lower optimal leverage than *E* for a constant level of active risk in the portfolio. More precisely, we see that as the distribution of risk shifts by a factor *k*, we should also expect the marginal change in leverage with respect to active risk (i.e the slope of the leverage curve against active risk) to shift by about 1/*k*.

[3] The appendix presents the general case with *N* assets.

What Type of Investment Processes Benefit More from Shorting?

In this section of the paper we explore the question of how different investment processes are affected by short extension implementation and which types of investment processes may benefit more from the additional flexibility associated with the transition from long-only to short extension portfolio construction.

Some long-only managers focus on picking "winning" stocks. Even these managers, who don't have forecasts for "losing" stocks, could benefit from relaxing the long-only constraint because shorting would release additional funding for larger long positions and would enable them to better cross-hedge long-position risk. However, such managers may not be able to take full advantage of the additional flexibility offered by long-short implementation. On the other hand, long-only managers that have symmetric alpha distributions may benefit more from long-short implementation. These managers have forecasts for both "winning" and "losing" stocks and long-short implementation enables them to express negative views more fully by shorting negative alpha stocks.

Among investment processes with symmetric alpha distributions, some produce forecasts for many stocks while others focus on a few stocks only. For example, a quantitative process that is based on a factor model will typically rank all stocks in the manager's universe. On the other hand, a fundamental process will typically focus on a relatively small number of stocks. Which process enjoys higher gain in information ratio from long-short implementation? At first sight, we may be tempted to think that processes with views on many stocks benefit more as optimal long-short portfolios exploit the entire alpha distribution. Careful analysis using our framework actually shows that this may not always be the case. In particular, for low levels of active risk, processes with views on few stocks achieve higher information ratio gains from short-extension implementation.

In order to gain insights into the relationship between alpha distribution and shorting, we write the optimal active weights as an orthogonal decomposition of risk-adjusted alphas:

$$\omega_k h_{A,k} = \eta \frac{\alpha_k}{\omega_k} + v_k$$

where ω_k is the residual risk of asset *k* relative to the benchmark and v_k's are residuals orthogonal to risk-adjusted alphas α_k / ω_k. Finally the η coefficient can be expressed as:

$$\eta = TC \frac{\sigma(\omega_k h_k)}{\sigma(\alpha_k / \omega_k)}$$

where *TC* is the transfer coefficient[4], $\sigma(\omega_k h_k)$ and $\sigma(\alpha_k / \omega_k)$ are the cross-sectional standard deviation of risk-adjusted active weights and risk-adjusted alphas in the portfolio. This orthogonal decomposition states that risk-adjusted active weights are nearly proportional to risk-adjusted alphas for well refined alphas and relatively small residuals.

[4] We define the transfer coefficient TC as the correlation between risk-adjusted weights and risk-adjusted alphas in the portfolio

Chart 4

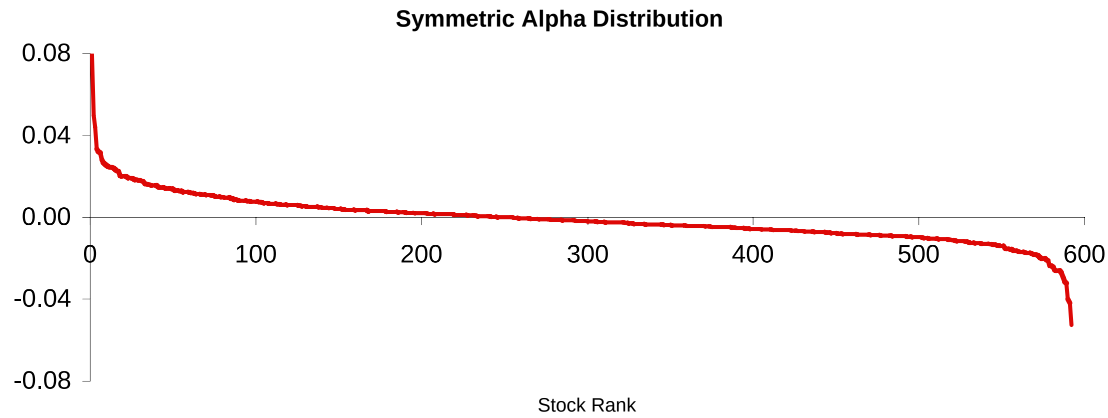


Chart 5

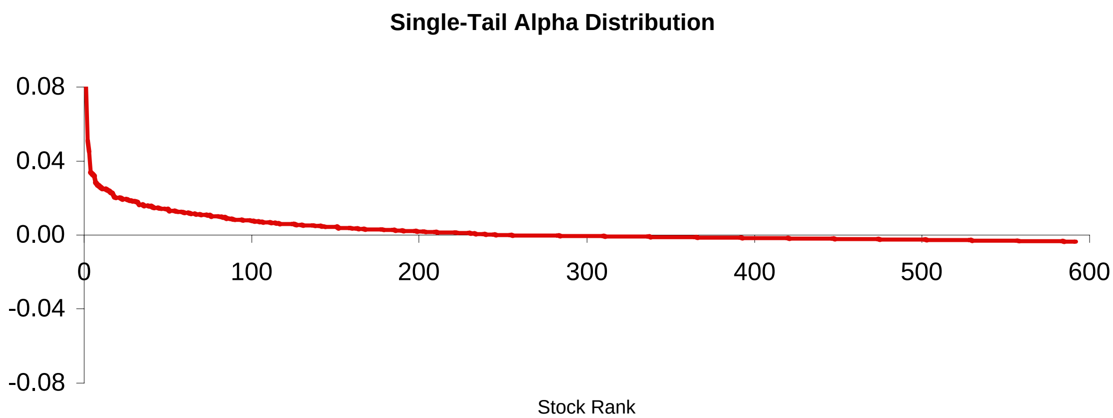
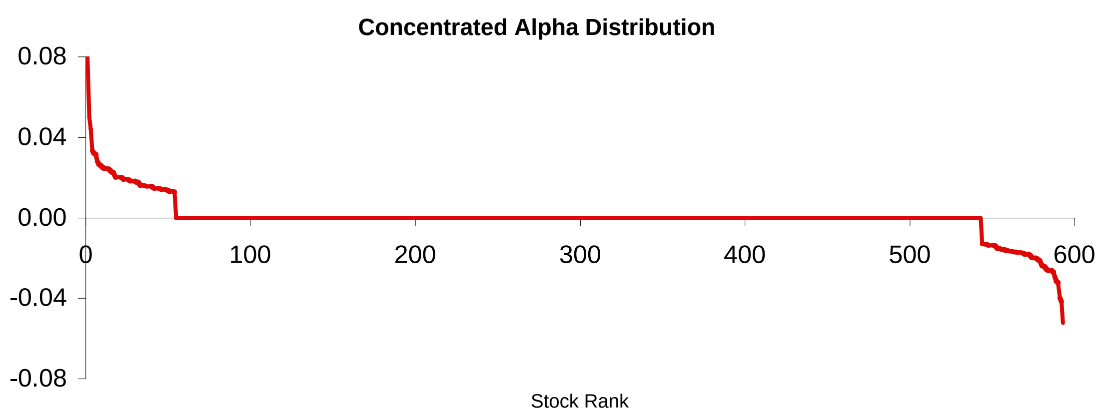


Chart 6



We will use our orthogonal decomposition of risk-adjusted active weights to illustrate the potential benefits of shorting for three different types of alpha distributions.

First, we consider a symmetric alpha distribution which would correspond to an investment process that can equally discriminate among winning and losing stocks. We can use a power function to model this type of distribution:

$$\alpha_k^1 = \begin{cases} (K/N)^{-1/\gamma}\left((k/k_0)^{-1/\gamma} - 1\right) & \text{for } k < k_0,\ K > 0,\ \gamma > 0 \\ -(K/N)^{-1/\gamma}\left(((N-k)/(N-k_0))^{-1/\gamma} - 1\right) & \text{for } k > k_0 \end{cases}$$

where N is the number of stocks in the selection universe, K and k_0 are scaling parameters while γ characterizes the thickness of the tails of the distribution. Chart 4 presents an example of this type of distribution, based on the EUE2L Barra Value Risk Index for the MSCI Europe Index constituents as of April 30, 2007. In this empirical alpha distribution, we found the power coefficient γ to be approximately equal to 1.8.

Second, we consider a single-tail alpha distribution, an example of which is depicted in Chart 5. This distribution could describe an investment process that focuses on selecting “winning” stocks and does not produce strong views regarding “losing” stocks. We assume these alphas decay exponentially in the left tail of the distribution and follow the same power function in the right tail of the distribution:

$$\alpha_k^2 = \begin{cases} (K/N)^{-1/\gamma}\left((k/k_0)^{-1/\gamma} - 1\right) & \text{for } k < k_0,\ K > 0,\ \gamma > 0 \\ \exp(-c(k-k_0)) - 1 & \text{for } k > k_0,\ c > 0 \text{ and } c << 1 \end{cases}$$

Third, we consider a concentrated alpha distribution, an example of which is depicted in Chart 6. This distribution could describe a fundamental bottom up investment process which produces alpha forecasts for a relatively small number of stocks in the manager’s universe:

$$\alpha_k^3 = \begin{cases} \alpha_k^1 & \text{for } k < k^* \\ 0 & \text{for } k^* \le k \le N - k^* \\ \alpha_k^1 & \text{for } k > N - k^* \end{cases}$$

The results reported in Table 4 suggest that investment processes that yield a symmetric alpha distribution experience higher gain in information ratio from relaxing the long-only constraint compared to investment processes that yield a single-tail alpha distribution. To understand why this conclusion holds generally, we compute the number of stocks N_1 and N_2 for which the long-only constraint binds for the two alpha distributions by using the orthogonal decomposition and plugging-in the corresponding alphas[5]:

[5] The appendix presents a formal derivation. We assume that the alphas are non-degenerate in the sense that they produce non-zero transfer coefficients and the residuals v_k are small in magnitude relative to risk-adjusted active weights (empirically we find that $v_k/\omega_k h_k$ <0.1).

$$N_1 = Card\left\{k_0 \le k \le N \mid k \ge N - \frac{N}{K}(N-k_0)\left(\frac{h_{B,k}\omega_k^2}{\eta}\right)^{-\gamma}\right\}$$

$$N_2 = Card\left\{k_0 \le k \le N \mid k \ge k_0 - \frac{1}{c}\ln(1-\frac{h_{B,k}\omega_k^2}{\eta})\right\}$$

where *Card* stands for the the number of elements in a given set. The symmetric alpha distribution leads to portfolios with a higher number of stocks that have negative total weights, and we see without ambiguity that N_1 will be much higher than N_2 as $\gamma > 0$ and $c \ll 1$. As these two alpha distributions only differ in the lower rankings, the number of stocks for which the long-only constraint binds reflects the gain in information ratio from relaxing the long-only constraint.

Table 4: Optimal Long-Only & Long-Short Portfolio Information Ratio for Different Alpha Distributions

	Symmetric Distribution			Single-Tail Distribution		
Active Risk	**Long-only IR**	**Long-short IR**	**% Gain in IR**	**Long-only IR**	**Long-short IR**	**% Gain in IR**
0.5%	0.82	0.89	9%	0.63	0.64	2%
1.0%	0.75	0.89	19%	0.60	0.64	7%
2.0%	0.64	0.89	39%	0.55	0.64	16%
4.0%	0.53	0.89	68%	0.48	0.64	33%
8.0%	0.41	0.89	117%	0.39	0.64	64%

Next, we explored the potential benefits from shorting for quantitative and fundamental investment processes through another case study. Two possible alpha distributions corresponding to these two types of investment processes are shown in Charts 4 and 6. These distributions cover all MSCI Europe Index constituents as of April 30, 2007, and were derived from asset exposures to the Barra EUE2L model Value Risk Index. The first distribution contains forecasts for all MSCI Europe Index constituents, while the second contains forecasts for the most extreme 100 rankings only (50 on each side). In this case study, we used the Barra EUE2L risk model to construct optimal portfolios for different levels of active risk and compared the gain in information ratio between long-only and long-short portfolios, based on these two different alpha distributions.

The results of this case study, reported in Table 5, suggest that the gain in information ratio between long-only and long-short portfolios for quantitative and fundamental investment processes may depend on the level of active risk, the number of forecasts in the alpha distribution, and the constant information ratio that could be achieved through unconstrained long-short optimization.

Table 5: Optimal Long-Only & Long-Short Portfolio Information Ratio for Different Alpha Distributions

	Views on All Stocks			Views on 100 Stocks		
Active Risk	**Long-only IR**	**Long-short IR**	**% Gain in IR**	**Long-only IR**	**Long-short IR**	**% Gain in IR**
0.5%	0.82	0.89	9%	0.69	0.79	15%
1.0%	0.75	0.89	19%	0.63	0.79	25%
2.0%	0.64	0.89	39%	0.57	0.79	39%
4.0%	0.53	0.89	68%	0.50	0.79	59%
8.0%	0.41	0.89	117%	0.41	0.79	94%

These results imply that, at low levels of active risk, investment processes with views on many stocks would experience lower gain in information ratio as a result of relaxing the long-only constraint, compared to processes that produce views on few stocks. A possible explanation could be that having views on many stocks leads to relatively high information ratio at low active risk levels, without the need for shorting. As a consequence, at low levels of active risk, the gain in information ratio from shorting is relatively low for processes with views on many stocks.

On the other hand, at high active risk levels, our results imply that processes with views on many stocks would enjoy higher gain in information ratio. This may be due to the fact that, at high levels of active risk, optimal long-only portfolios are concentrated in the most extremely ranked stocks, irrespective of the number of forecasts in the alpha distribution. On the other hand, optimal long-short portfolios exploit the entire alpha distribution; therefore having views on many stocks may lead to higher information ratio in long-short portfolios. As a consequence, at high levels of active risk, the gain in information ratio from relaxing the long-only constraint may be higher for processes with views on many stocks.

Our results are consistent with the standard practice that many quantitative managers follow of running long-only portfolios at relatively low active risk levels (e.g. enhanced index strategies) as they may be able to achieve relatively high information ratio at low active risk levels, without the need for shorting. On the other hand, our results suggest that shorting flexibility at high active risk levels may enable quantitative managers to better exploit the entire alpha distribution and achieve higher gains in information ratio.

Now we use our orthogonal decomposition to show why the conclusions we reached through this case study are also robust to more general assumptions. Again, we compare the number of stocks for which the long-only constraint is binding for the two alpha distributions:

$$N_1 = Card\left\{k_0 \le k \le N \;\middle|\; k \ge N - \frac{N}{K}(N-k_0)\left(\frac{h_{B,k}\omega_k^2}{\eta}\right)^{-\gamma}\right\}$$

$$N_3 = Card\left\{k_0 \le k \le N \;\middle|\; k \ge N - \frac{N}{K}(N-k_0)\left(\frac{h_{B,k}\omega_k^2}{\eta}\right)^{-\gamma} \; and \; k > N - k^*\right\}$$

As mentioned previously, for a given alpha distribution, the number of stocks for which the long-only constraint binds reflects the gain in information ratio from relaxing the long-only constraint. We see that N_1 and N_3 may be close for low levels of active risk as η is increasing in active risk. In this case, the gain in IR from shorting will be higher for the concentrated alpha distribution as the long-only portfolio IR of the concentrated alpha distribution is lower compared to the long-only portfolio IR of the alpha distribution with views on all stocks, for a given level of active risk. However, there will be a threshold level of active risk above which N_1 will eventually be higher than N_3 as N_3 cannot be greater than k^*. Thus only for high levels of active risk will the gain in *IR* from shorting for strategies with views on all stocks be higher than for strategies with views on a small number of stocks.

Conclusion

One of the main challenges in short extension portfolio construction is to determine the appropriate level of leverage. In this paper, we analyzed the relationship between portfolio leverage and active risk and demonstrated analytically and empirically that there is an optimal level of leverage for a given level of active risk. This conclusion could have significant implications for the implementation of short extension portfolios. The main implication is that leverage and active risk should not be determined independently in short extension portfolios. Upper limits on leverage may be imposed for various reasons (for example, regulatory or client specific requirements), however, our analysis shows that such restrictions may reduce the information ratio of the portfolio.

Changing market conditions and different investment universes will generally imply different levels of volatility for the constituents of the portfolio. In turn, this parameter affects the optimal level of leverage required in a short extension portfolio for a given level of active risk. In particular, lower volatilities lead to higher optimal leverage for a given level of active risk. Higher required leverage for a given level of active risk implies higher gain in information ratio from relaxing the long-only constraint. These findings could have potentially interesting implications for long-only managers who are considering launching 130/30 strategies, as well as 130/30 managers who are considering applying their investment process to another investment universe with different volatility characteristics.

Finally, we examined how different investment processes may be affected by short extension implementation. We found that symmetric alpha distributions lead to higher gain in information ratio from relaxing the long-only constraint compared to single-tail alpha distributions. Also, we demonstrated that investment processes that are able to rank all stocks in the manager's universe achieve higher gain in information ratio at high levels of active risk, compared to concentrated investment processes that produce alpha forecasts for a relatively small number of stocks. These results suggest that shorting flexibility at high active risk levels may enable quantitative managers to better exploit the entire alpha distribution and achieve higher gains in information ratio.

Appendix

A1. Computing Optimal Leverage

Recall the unconstrained long-short optimization problem:

$$\max_{h_A}\left\{h_A'\alpha-\frac{1}{2}\lambda h_A'Vh_A\right\}$$
$$Subject\ to:\ h_A'e=0$$

The Lagrangian for this problem can be written as:

$$L=h_A'\alpha-\frac{1}{2}\lambda h_A'Vh_A-\mu h_A'e$$

where μ is the Lagrange multiplier associated with the budget constraint. The first-order conditions for optimality can be expressed as:

$$\begin{cases}\dfrac{\partial L}{\partial h_A'}=\alpha-\lambda Vh_A-\mu e=0\\ \dfrac{\partial L}{\partial \mu}=h_A'e=0\end{cases}$$

The optimal active weights are then given by:

$$h_A=\frac{\sigma_A}{\left(\alpha'V^{-1}\alpha-\dfrac{(e'V^{-1}\alpha)^2}{e'V^{-1}e}\right)^{1/2}}V^{-1}(\alpha-\frac{e'V^{-1}\alpha}{e'V^{-1}e}e)$$

where σ_A is the portfolio active risk:

$$\sigma_A=(h_A'V\,h_A)^{1/2}=\frac{1}{\lambda}\left(\alpha'V^{-1}\alpha-\frac{(e'V^{-1}\alpha)^2}{e'V^{-1}e}\right)^{1/2}$$

The optimal level of leverage for the unconstrained long-short portfolio which corresponds to the above optimal active weights is given by the following expression:

$$
\begin{aligned}
L^* &= \sum_{i=1}^{N} \left| h_{A,i} + h_{B,i} \right| - 1 \\
&= \frac{\sigma_A}{\left(\alpha' V^{-1} \alpha - \frac{\left(e' V^{-1} \alpha\right)^2}{e' V^{-1} e} \right)^{1/2}} \varepsilon' V^{-1} \left(\alpha - \frac{e' V^{-1} \alpha}{e' V^{-1} e} e\right) + \varepsilon' h_B - 1 \\
&= \frac{\sigma_A}{\left(\alpha' V^{-1} \alpha - \frac{\left(e' V^{-1} \alpha\right)^2}{e' V^{-1} e} \right)^{1/2}} \sum_{i=1}^{N} \varepsilon_i \left(V^{-1} \left(\alpha - \frac{e' V^{-1} \alpha}{e' V^{-1} e} e\right) \right)_i + \varepsilon' h_B - 1
\end{aligned}
$$

where ε is a $N \times 1$ vector defined as:

$$
\begin{aligned}
\varepsilon_i &= 1 \quad if \;\; h_{A,i} + h_{B,i} \geq 0 \\
\varepsilon_i &= -1 \; if \;\; h_{A,i} + h_{B,i} < 0
\end{aligned}
$$

A2. Imposing Constraints on Leverage

Now we solve the optimal long-short portfolio problem with an active risk target of σ_A:

$$\max_{h_A}\left\{h_A'\alpha-\frac{1}{2}\lambda h_A'Vh_A\right\}$$

$$\begin{aligned}&\textit{Subject to :}\\&(i)\quad h_A'e=0\\&(ii)\ \ h_A'Vh_A=\sigma_A^{\,2}\end{aligned}$$

The Lagrangian now becomes:

$$L=h_A'\alpha-\frac{1}{2}\lambda\sigma_A^{\,2}-\mu h_A'e-\gamma(h_A'Vh_A-\sigma_A^{\,2})$$

And the first-order conditions for optimality are now expressed as:

$$\begin{cases}\dfrac{\partial L}{\partial h_A'}=\alpha-\mu e-2\gamma Vh_A=0\\[2mm]\dfrac{\partial L}{\partial \mu}=h_A'e=0\\[2mm]\dfrac{\partial L}{\partial \gamma}=\sigma_A^{\,2}-h_A'Vh_A=0\end{cases}$$

When expressed as a function of active risk, the optimal active weight and the optimal leverage remain unchanged compared to the unconstrained case. In particular, as in the previous case, there is a unique optimal leverage associated with the target active risk level. Therefore, imposing an additional leverage constraint will lead to a decrease in information ratio unless the leverage target is equal to the optimal leverage level. Likewise, imposing an active risk target in addition to a leverage target will also decrease the information ratio unless the target active risk is equal to the optimal active risk associated with the leverage target.

A3. Orthogonal Decomposition of Risk-adjusted Active Weights

Recall the orthogonal decomposition of risk-adjusted optimal active weights:

$$\omega_k h_{A,k} = \eta \frac{\alpha_k}{\omega_k} + v_k$$

where ω_k is the residual risk of asset *k* relative to the benchmark and v_k's are residuals orthogonal to risk-adjusted alphas α_k / ω_k. The *η* coefficient can be expressed as:

$$\eta = TC \frac{\sigma(\omega_k h_k)}{\sigma(\alpha_k / \omega_k)}$$

where *TC* is the transfer coefficient defined as the correlation between risk-adjusted active weights and risk-adjusted alphas, $\sigma(\omega_k h_k)$ and $\sigma(\alpha_k / \omega_k)$ are the cross-sectional standard deviation of risk-adjusted active weights and risk-adjusted alphas in the portfolio. We further assume that the following two conditions hold:

- $TC \neq 0$ and $TC > 0$
- $|v_k| / |\omega_k h_{A,k}| \approx 0$ for small *k*'s and large *k*'s (extreme rankings in the alpha distribution)

The first condition states that risk-adjusted active weights should reflect risk-adjusted alphas, while the second condition states that risk-adjusted active weights should be nearly proportional to risk-adjusted active weights in both ends of the alpha distribution (empirically we find that $|v_k| / |\omega_k h_{A,k}| < 0.1$).

We can obtain a formal justification of the fact that risk-adjusted optimal active weights are nearly proportional to risk-adjusted alphas as follows. Recall the expression for the unconstrained long-short optimal active weight, omitting the budget constraint:

$$h_A = \frac{\sigma_A}{(\alpha' V^{-1} \alpha)^{1/2}} V^{-1} \alpha$$

Decompose returns into a benchmark component and a residual component:

$$r = \beta r_b + w$$

where r_b is the benchmark return, β is the vector of betas with respect to the benchmark, and *w* denotes the residual return. Thus the asset covariance matrix can be expressed as:

$$V = \beta \beta^T \sigma_b^2 + \Omega$$

Where σ_b^2 is the benchmark volatility and Ω is the covariance matrix of residual returns. Using matrix algebra, we can write the inverse of the asset covariance matrix as:

$$V^{-1} = \Omega^{-1} - \frac{\Omega^{-1} \beta \beta^T \Omega^{-1}}{\beta^T \Omega^{-1} \beta + 1/\sigma_b^2}$$

If we make the assumption that risk-adjusted alphas ($\Omega^{-1}\alpha$) are nearly orthogonal to betas ($\beta^T(\Omega^{-1}\alpha)=0$), we can then rewrite the optimal active weights as:

$$h_A = \frac{\sigma_A}{(\alpha'\Omega^{-1}\alpha)^{1/2}}\Omega^{-1}\alpha$$

If we further make the assumption that residual returns are uncorrelated with each other, Ω^{-1} becomes a diagonal matrix and risk-adjusted active weights ($\omega_k h_{A,k}$) are proportional to risk adjusted alphas (α_k / ω_k):

$$\omega_k h_{A,k} = \frac{\sigma_A}{\left(\sum_{j=1}^{N}\frac{\alpha_j^{\,2}}{\omega_j^2}\right)^{1/2}}\frac{\alpha_k}{\omega_k} = \frac{\left(h_A' V h_A\right)^{1/2}}{N\sigma\left(\alpha_k/\omega_k\right)}\frac{\alpha_k}{\omega_k} = \frac{\left(h_A'\Omega h_A\right)^{1/2}}{N\sigma\left(\alpha_k/\omega_k\right)}\frac{\alpha_k}{\omega_k} = \frac{\sigma\left(\omega_k h_k\right)}{\sigma\left(\alpha_k/\omega_k\right)}\frac{\alpha_k}{\omega_k}$$

Note that in this case, TC would be equal to one. Residual returns will be uncorrelated when the benchmark embeds the true undiversifiable component of the cross-section of asset returns. In general, risk-adjusted alphas will show some correlation with benchmark betas and the budget constraint as well as other types of constraints (leverage, transaction costs, etc) will affect optimal portfolio weights. This will lead to a TC which will be below one and non-zero residual terms v in the relationship between risk-adjusted active weights and risk-adjusted alphas. The above derivation can be seen as an approximation to our more general orthogonal decomposition.

A4. Computing the Number of Binding Constraints

Given our orthogonal decomposition, we can compute an approximation for the number of stocks with strictly negative total weights for each of the symmetric, single-tail and concentrated alpha distributions we consider.

Note that for the power law alpha ranking such as the symmetric alpha, we can assume that $\left(\frac{N-k}{N-k_0}\right)^{-1/\gamma}$ is much greater than 1 for the extreme low rankings. Equivalently, we have that:

$$\Pr(\alpha_k < -u) = \frac{N-k_0}{K} u^{-\gamma} \text{ for large } u\text{'s}$$

Using our orthogonal decomposition with the fact that residuals are small for the extreme bottom rankings, an approximation for the number of stocks with strictly negative weights for the symmetric alpha ranking is:

$$N_1 \approx N \times \Pr(h_{A,k} + h_{B,k} < 0) = \frac{N}{K}(N-k_0)\left(\frac{h_{B,k}\omega_k^2}{\eta}\right)^{-\gamma}$$

which can also be expressed as:

$$N_1 = Card\left\{k_0 \le k \le N \;\middle|\; k \ge N - \frac{N}{K}(N-k_0)\left(\frac{h_{B,k}\omega_k}{\eta}\right)^{-\gamma}\right\}$$

For the single-tail alpha distribution, the number of binding long-only constraints N_2 can be obtained by plugging the expression for the bottom rankings in the orthogonal decomposition. Finally, for the concentrated alpha distribution, the derivation is similar to the symmetric ranking case except for taking into account that the concentrated ranking is only non zero for the last *k** assets.

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