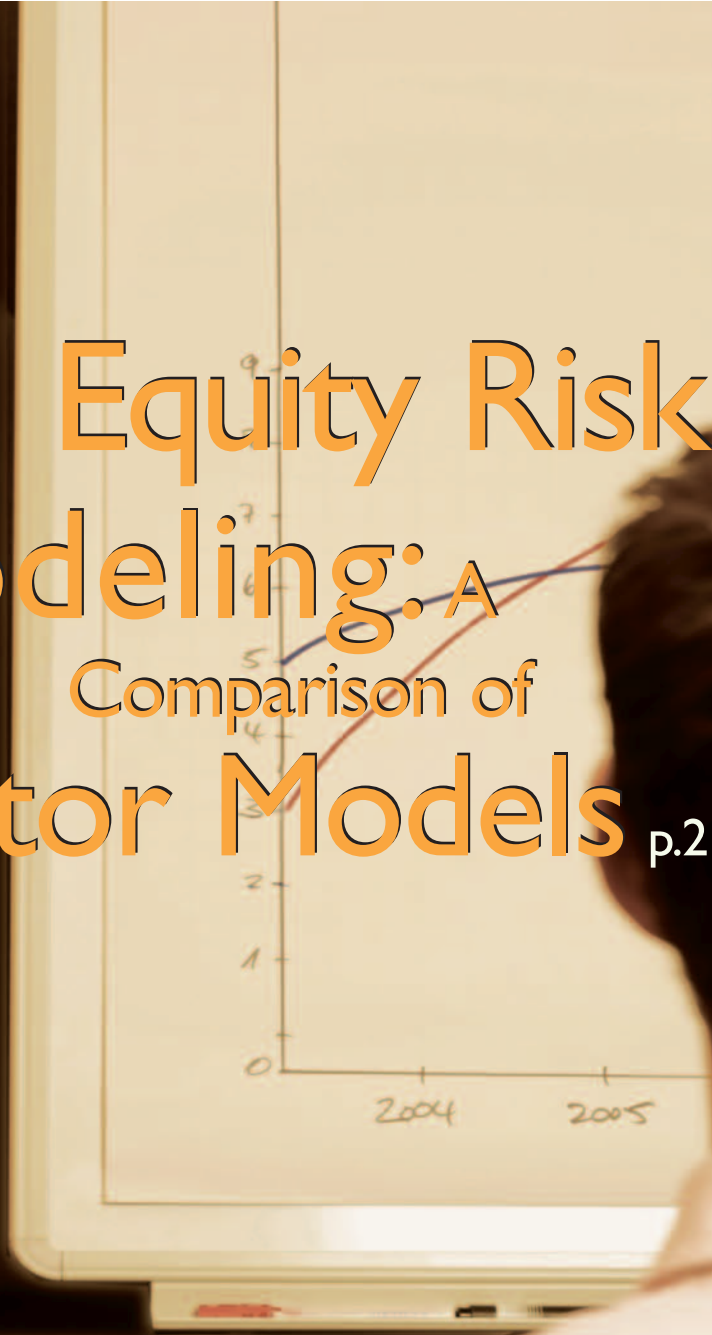




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# HORIZON

THE MSCI BARRA NEWSLETTER

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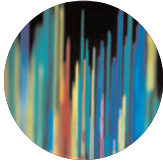
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## Welcome to MSCI Barra Horizon.

In this issue of Horizon we cover a broad spectrum of topics, highlighting the diversity and depth of MSCI Barra's research across equity, international fixed income, and credit default spread modeling.

The first paper by Guy Miller, Executive Director, Analytics Research, compares Fundamental, Statistical, and 'Hybrid' Equity Factor Risk models. He discusses when the different types work best and when they are likely to fail in risk management and portfolio construction. When statistical factors are used to extend a fundamental factor model, we see modest improvements in risk forecasting. The improvement in portfolio optimization seems even slighter and should be applied only with caution.

The second article by Lisa Goldberg, Executive Director, Analytics Research, Rajnish Kamat, Vice President, Analytics Research, and Vijay Poduri, Vice President, Analytics Research, assesses the effectiveness of the Barra Default Probability (BDP) model in explaining the cross-sectional variation of Credit Default Spreads. In order to establish the usefulness of the BDP model in forecasting real-world defaults, we test it against historical default experience. We find that the model shows good default discriminatory power relative to agency ratings.

Next, Oren Cheyette and Boris Postler, Senior Associate, Analytics Research, introduce an Implied Prepayment model to calculate spread values and effective Durations for Danish Mortgage Backed Bonds (MBB). By using an implied prepayment model, constructed by fitting a generic functional form to market prices of liquid Danish MBB, we take the market price of prepayment risk into account and produce consistent results. Since only current pricing data are used as a model input this approach does not require access to a historical database of prepayment data. This will allow us to extend the model to cover different markets with comparative ease.

Finally, Frank Nielsen, Executive Director, Research Consulting, continues the discussion on the implications of changing volatility levels on active and passive portfolio management. In the Summer 2005 Horizon Newsletters, we examined the sources of cross-sectional volatility in the Japan market. We extend the study to Europe and the US market and simulate the impact of dynamically changing volatility levels on active portfolio risk. We show that the optimal level of tracking error, the size of active exposures, and the optimal number of securities vary wildly with changing volatility levels.

Horizon regularly features a listing of publications by MSCI Barra's research group.

Horizon is available on the web at <http://www.barra.com/news/horizons.aspx>.

*Frank Nielsen*

Frank Nielsen, CFA, MSCI Barra



# Equity Risk Modeling: A Comparison of Factor Models

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## An Introduction to Factor Models

Portfolio managers know that return comes at the price of risk. They also know that some forms of risk are less rewarding than others. While pursuing return they therefore attempt to control risk by diversification and hedging.

Diversifiable risk is fundamentally different from risk that must be hedged. Diversifiable risk arises from sources of return that are independent from one company to another. To reduce the amount of diversifiable risk in a portfolio, a manager can increase the number of companies held. Nondiversifiable risk, sometimes called common factor risk, is more difficult to manage - although it is often the dominant contributor to risk at the portfolio level. Market risk, in which all assets move up or down together, is the most familiar example of common factor risk. Other examples include risk shared among members of a common industry, or risk common to all portfolios with a value tilt.

Hedging is a quantitative problem. If a manager wants to eliminate market risk, or the risk associated with a style characteristic such as value, what set of trades will be most effective? Can assets in the energy sector counterbalance positions in the airlines industry or in transportation—and if so, in what proportions?

Factor models of risk help answer these questions. Factor models identify sources of nondiversifiable risk called *factors*, and quantify the sensitivity or *exposure* of each asset to each source. More precisely, a factor model takes the return to each asset and decomposes it into a diversifiable piece called the *specific return* and contributions from the common factors. The attribution equation

$$r = Xf + s$$

expresses this decomposition mathematically. It rewrites asset returns  $r$  as the sum of factor contributions  $Xf$  and specific returns  $s$ . The contribution from each factor is the product of a factor return  $f$  and an exposure  $x$ . Consider the return to an asset ABC in a model that contains only two factors, a market factor and a value factor. The model attributes the return to ABC as  $r_{ABC} = X_{ABC, Market} f_{Market} + X_{ABC, Value} f_{Value} + s_{ABC}$ .

Suppose that ABC has a high value exposure and we wish to overweight it as part of a value tilt strategy, but that ABC also carries a high market exposure. We do not want to take on additional market risk. The factor model can help us achieve the desired portfolio market exposure by identifying other assets with low value exposures and high market exposures. We can eliminate unwanted market risk while enhancing our value tilt by underweighting them in the correct proportions.

In addition to guiding portfolio construction, attribution plays an essential role in performance analysis. Decomposing portfolio returns into their component sources can show a manager whether a strategy is performing as expected, or whether returns actually come from unanticipated and potentially dangerous sources.

For both portfolio selection and performance attribution, quantitative assessments of risk are invaluable. A risk model enables a portfolio manager to construct efficient bets that optimize the return expected from a given risk budget.

Factor models facilitate risk forecasting by identifying a relatively small number of sources of common risk. Without factor models, forecasting risk would involve characterizing statistical returns relationships between every pair of assets in the market. This is a vast number of individual relationships—over half a million for a universe of 1000 assets. Factor models divide the risk forecast into two independent parts, a common factor risk forecast and a forecast of diversifiable risk for each asset. The common factor forecast itself consists of two tasks: calculating the exposures of each asset to the model factors, and forecasting the statistical relationships (i.e., the covariances) between factor returns. A model with 1000 assets and 50 factors requires 1,000 asset-level specific risk forecasts, 50,000 exposures of assets to the factors, and 1,275 factor return covariances. In total there are 52,275 quantities, about one-tenth of the half million quantities needed by a model without the factor structure. An unstructured model without factors applies its input data ineffectively. It expends much of the data quantifying irrelevant relations. A factor model can apply the same input data far more efficiently by selecting, from among all the conceivable relationships between asset returns, only those that appear likely to influence future

risk. It excludes the majority of potential relationships as spurious, and does not waste data quantifying them.

### Types of Factors

Factors in common use fall into several categories. Although finer distinctions are possible, this paper distinguishes three broad factor types (see Connor 1995). The classification scheme depends on how factor returns and exposures are calculated.

A *fundamental factor* model assigns factor exposures at the beginning of a returns period, for example the month of March. The exposure assignments are based on fundamental financial characteristics such as industry membership, capitalization, financial ratios like book-to-price, and possibly other items like historical returns volatility. At the end of the month, a cross-sectional regression reveals the set of factor returns that best account for the observed asset-level returns over the month. The fundamental model thus assigns exposures *ex ante* and estimates factor returns *ex post*.

In *macroeconomic factor models*, the pattern is reversed. Instead of assigning exposures, a macroeconomic model assigns factor returns. The factor returns may be the returns to major equity indices, to the prices of commodities like oil, to interest rates, or to other economic drivers. The exposures of each asset to the factors are estimated by comparing the historical asset returns with historical factor returns in a time-series regression. Thus, at the beginning of April an asset's exposures to the model factors depend on its returns and factor returns from March and before.

*Statistical factor models* look for patterns of commonality in the historical record of asset returns. From a history of  $T$  returns to each of  $N$  assets in the model universe, the statistical model simultaneously extracts a history of factor returns (the number of factors is chosen to

be far less than the number of assets) and estimates the exposure of each asset to each factor. The usual method for the estimation is called *principal components analysis*, or PCA.

To summarize, fundamental models use asset returns over a single period to estimate the factor returns over that period, macroeconomic models use a history of asset returns to estimate asset exposures to the factors (presumably good for the full history and for the first period following the history), and statistical models extract both factor returns and exposures from historical asset returns. Factor models in actual use sometimes blend or hybridize these three approaches.

Each of the three factor model types differ in terms of what inputs are demanded and what is offered in exchange. All require a history of asset returns—but assets with little or no returns history can be treated more consistently in fundamental models than in the other two types, which necessarily experience difficulty assigning exposures to such assets. In addition to the returns history, fundamental models typically require detailed industry, financial, and trading data for each asset. Macroeconomic models are less demanding; in addition to the returns history they need only historical returns to the model factors. Statistical models make do with the returns history alone. As might be expected, the burden imposed by data collection and cleaning is offset by what each model type offers in terms of structure and ease of interpretation.

What are the advantages and disadvantages of each modeling approach? A detailed answer depends on how the factor model is to be applied, but in the broadest terms the issue reduces to the relative importance of specification error versus measurement error.

By building its factor exposures directly from financial data, a fundamental model imposes a more sharply defined and rigid structure on returns than a macroeconomic or statistical

model. If the exposures are badly constructed, or if a factor is omitted from the model its performance suffers. A “well built” fundamental model represents a significant effort to avoid such specification errors.

At the other extreme, a statistical model makes no explicit assumptions about the asset characteristics that drive returns commonality. It lets patterns in the returns history speak for themselves, and no pattern or possible factor is excluded *a priori*. This might seem to be the perfect formula for success, but statistical models can fail to capture financially significant factors. The failure occurs precisely because of the lack of imposed structure. In any returns history many relations between asset returns will appear. A few express underlying connections between assets that will persist in the future, but coincidental relationships form the vast majority. Statistical models must select factors from the realm of possibilities offered by the returns history, and then estimate the exposure of each asset to each factor statistically. Statistical modelers deal with this problem by admitting only strong relationships in their models, as these seem least likely to be spurious. Nevertheless, the sheer number of accidental relationships is so large that statistical factors are always admixtures of accident and mechanism. Typically the measurement errors in estimated exposures are large, causing the model to miss some sources of risk and misattribute others.

So which is more likely, that specification errors in a carefully constructed fundamental model will cause it to misconstrue risk, or that measurement error in a statistical model will make its factors too inaccurate to use? This paper explores this question in two phases. First, it asks what factors a statistical model can find and usefully exploit. Second, it looks for evidence of misspecification in a carefully constructed fundamental factor model, MSCI Barra's Japanese equity risk model JPE3. It omits discussion of macroeconomic factor

models, which suffer from both specification error and measurement error and are generally felt to be less successful than either fundamental or statistical models (see e.g., Connor 1995).

### Successful and Unsuccessful Factor Hunts

In principle, a fundamental factor model can find and exploit any common factor that can contribute significantly to a portfolio's risk. The reason is that the factor returns in fundamental models are the returns to *factor portfolios*. A factor portfolio is the most diversified portfolio possible that is exposed only to the factor in question; it typically combines long and short positions. The factor's presence is verified if the return to its factor portfolio differs significantly from the expected return under pure diversification. If the factor is too weak to manifest itself in the returns to the factor portfolio, it is also too weak to appear significantly in the returns to any other portfolio.

The problem of course is that to look for a fundamental factor, one needs to know how to construct its factor portfolio. Factor exposures are the essential ingredients for this construction. While it is true that fundamental factor models can find any financially significant factor, they have to be told where to look. Statistical factor models do not need to be told where to look. They uncover factors by themselves, searching for patterns in historical returns.

Having to search without guidance makes a statistical model less sensitive than a fundamental model. If a fundamental model detects an apparent factor in the portfolio where it was expected, it is likely the factor is real and not the manifestation of a rare statistical accident. On the other hand a statistical model searches a much larger space of possible factor patterns, and so the likelihood of a statistical accident producing the appearance of a factor is much higher. The statistical model's detection

threshold must be set high enough to avoid spurious patterns, and the model consequently loses sensitivity. The fundamental model purchases its higher sensitivity by applying additional information (e.g., industry membership data) to the search.

### A Simple Pattern Hunt

To develop intuition about unguided searches, it is helpful to think about a simple pattern-finding problem. Imagine a pattern-hunter sitting in the bleachers of a sports stadium, looking for patterns on the field below. On the field is a crowd of 1,000 people. Every minute, each person in the crowd will choose either a red or a blue card to hold up and display. Some people in the crowd choose colors at random. Others follow a program that specifies which card to hold up each minute. At any one time all of the people who follow the program therefore hold up the same color, although the particular color can change from moment to moment. How difficult is it to securely identify the program and the "conspirators" who follow it?

The answer depends on how many conspirators there are. If 800 people follow the program, it should not take long to (1) notice that a conspiracy exists and (2) decide who is involved. On the other hand, if the conspiracy involves only 20 people, just noticing the conspiracy's existence could take a long time. A more extensive pattern is easier to find.

The answer also depends on how persistent the pattern is. If the program includes "breaks" during which conspirators are free to choose colors at random, or if the program lasts for only a short time, the pattern will be much more difficult to identify.

This simple example leaves out one important feature of pattern-finding. Since the conspirators in the example follow the program without fail, the pattern is very strong. One can

imagine a weaker pattern in which individual conspirators can occasionally “defect” from the program and select a color at random.

Obviously, weaker patterns will resist identification longer than strong ones.

To summarize, patterns are easier to find if they are extensive, persistent, and strong. To find patterns that are narrow, intermittent, or weak is difficult; such patterns can only be found if the data history offers many instances to study. Short-lived patterns can be impossible to find.

### *Hunting for Financial Factors*

Turn now from the “conspirator search” to a more realistic example of a statistical search for a hidden financial factor. A hunt for financial factors contains all the elements of the conspirator search. Consider looking for a single factor hidden in a universe of  $N$  assets. Of these, are  $n$  exposed to the factor; suppose that their exposures are equal to 1. The remaining assets have exposures  $X=0$ . (Here the non-zero exposures are set to 1 for simplicity. The results will remain valid if the exposed assets have an average squared exposure equal to 1.) Over every returns period  $t$ , each asset  $a$  produces a return  $r_{at} = X_a f_t + s_{at}$ . The factor can be either active or inactive in a given time period. When the factor is active, the factor return  $f_t$  is drawn randomly from a distribution with mean 0 and standard deviation  $\sigma_f$ . When the factor is inactive, the factor return is simply set equal to 0. The total history of returns contains  $T$  periods; the factor is active over  $\tau$  of them. The specific return  $s_{at}$  for each asset is drawn from another distribution with mean 0 and standard deviation  $\sigma_s$ ; all assets in this simplified world have the same specific risk. This framework for a financial pattern search contains all of the essential ingredients: the breadth of the factor depends on the ratio  $n/N$ , its persistence on  $\tau/T$ , and its strength on the variance ratio

$\sigma_f^2/\sigma_s^2$ . The search space itself is described by the number of assets  $N$  and the history length  $T$ .

In the conspirator search, success is defined as identifying the conspirators and the program of card choices with reasonable confidence.

When searching for a financial factor, success is determined by the accuracy of the factor exposures and the factor returns that the search recovers. Financial decision-making requires accurate exposures; inaccurate exposures cannot guide effective hedges. Furthermore, if the factor returns are poorly known their risk cannot be estimated well. To make the requirements more concrete, suppose that the statistically recovered factor exposures  $\hat{X}$  differ from the correct exposures  $X$  by errors  $\epsilon$ :  $\hat{X} = X + \epsilon$ . Similarly, the statistically recovered factor returns  $\hat{f}$  differ from the true factor returns by  $f$  errors  $\delta$ . A natural definition of success is that errors constitute less than half of the statistical exposures and of the factor returns:  $\langle X^2 \rangle > \langle \epsilon^2 \rangle$  and  $\langle f^2 \rangle > \langle \delta^2 \rangle$ . (The angle brackets here represent averages over assets and averages over time periods in the history, respectively.)

A statistical model can recover factor returns and exposures with acceptable accuracy if the factor meets the success criterion

$$\min(T, N) \frac{\tau}{T} \frac{n}{N} \frac{\sigma_f^2}{\sigma_s^2} > 1$$

The success criterion is simply a quantitative restatement of the qualitative lessons from the conspirator search. It says that factors which are weak, or narrow in time or in cross-section, are hard to find.

For the mathematically inclined, the success criterion is not difficult to derive. Assume that half of the statistical model's work is done and that factor returns are already known.

Regressing the history of an asset's returns against the factor returns yields its estimated

factor exposure  $\hat{X} = X \pm (\sigma_s / \sigma_f) \sqrt{1/\tau}$

Demanding that on average less than half of the estimated factor exposure be error (noise) results in the condition

$\tau \frac{n}{N} \frac{\sigma_f^2}{\sigma_s^2} > 1$ . (Note that this condition would also arise in calculating exposures within the context of a macroeconomic factor model.

Macroeconomic factors also suffer from estimation error.) If on the other hand the exposures were known and the factor returns had to be recovered through a regression, clean factor returns would require  $\tau \frac{n}{N} \frac{\sigma_f^2}{\sigma_s^2} > 1$ . Combining these two requirements produces the success criterion (1.2).

Note that the success criterion is based on very basic statistical considerations, and does not depend on details such as whether or not the specific returns are normally distributed, or how the statistical factor model is implemented. To the extent that reality departs from the idealized picture used to derive the criterion—the factor strength might vary over time, the specific return size might vary across assets, etc.—a statistical factor search is more likely to fail than the criterion indicates.

### Typical Numbers and their Implications

To understand the implications of the success criterion, we need to know how the numbers work out in standard financial contexts. In most financial applications, the number of assets under consideration exceeds the number of relevant time periods available for study, so  $\min(T, N) = T$ . As a particular example, if two years of weekly data are used to evaluate the risk of portfolios selected from a field of 500 assets, then  $T=104$ ,  $N=500$  and  $\min(T, N)=104$ . A persistent factor has a time-coverage  $\tau/T$  close to 1. An intermittent or short-lived factor would have a smaller time-coverage. An extensively exposed factor has an asset-cover-

age  $n/N$  close to 1. A narrowly exposed factor would have a smaller asset-coverage. For example, a factor operating only in a single GICS sector might have a coverage  $n/N \approx 0.1$ . The ratio  $\sigma_f^2 / \sigma_s^2$  of factor return variance to specific return variance is typically quite small, but is larger for stronger factors. The strongest single factor in a market is the market factor itself, for which the variance ratio might be as large as 0.3. Other factors are much weaker; a strong style factor might have a variance ratio of a few percent - good values for purposes of illustration might be  $\sigma_f^2 / \sigma_s^2 \approx 0.02$  or 0.03.

These numbers imply that a statistical search using 2 years of weekly data can successfully find a strong, broad style factor: invoking the success criterion (1.2) as a test one finds  $104 \times 1 \times 1 \times 0.02 > 1$ . On the other hand, a strong but recent or short-lived style factor would be more marginal. If the factor has existed only over the past year then  $\tau/T \approx 0.5$  and the success test yields a borderline result:  $104 \times 0.5 \times 1 \times 0.02 > 1$ . Factors confined to individual industrial sectors would be invisible to a statistical search:  $104 \times 1 \times 0.1 \times 0.02 < 1$ . Purely statistical methods are ill-suited to finding needles in haystacks, whether the figurative haystack is in time or in the multitude of assets.

Another implication of the numbers is that it is difficult to find any factor other than the market with monthly returns. A strong, broad style factor just becomes visible with a search based on 5 years of monthly returns. Searches based on higher frequency returns are more likely to succeed.

### Computer Simulations of Statistical Factor Searches

Computer simulations offer a more detailed look into the performance of statistical factor searches than the simple success criterion.

The computer prepares a history of  $T$  returns for each of  $N$  assets. In each of the  $T$  periods, the computer randomly generates  $N$  independent specific returns and one factor return. From these, it calculates asset returns according to the attribution formula (1.1). Following the simple financial pattern search example described above, the first  $n$  of the  $N$  assets are exposed to the factor with exposures  $X=1$ ; the remaining assets are unexposed and have exposures  $X=0$ . A statistical factor model examines the simulated returns history and tries to determine the values of the exposures and factor returns. The success of the statistical search can be evaluated by comparing the statistically recovered exposures with the true exposures. The spirit of the computer simulations is akin to testing “code-breaking” software by encoding a secret message, then giving it to the software to see how well it recovers the message.

Figure 1 shows the result of a simulated factor search in a field of 1,000 assets. Of these, 100 are exposed to a hidden factor. The search works from a history of 100 periods: equivalent to a half-year of daily data, two years of weekly data, or eight years of monthly data. To ensure that the search is at least marginally successful, the hidden factor is made unrealistically strong; its variance ratio is  $\sigma_f^2 / \sigma_s^2 = 0.1$ .

Two histograms appear in Figure 1. The left-most histogram describes the distribution of statistically recovered exposures in the population of the 900 unexposed assets. Ideally, the factor search would have discovered that all these assets have exposures equal to 0. Instead, the statistical search recovered values scattered around an average of 0. The bulk of the recovered values falls between -0.7 and 0.7. The histogram on the right shows the distribution of statistically recovered exposures for assets exposed to the hidden factor. The sta-

tistically estimated exposures typically lie between 0.3 and 1.7; the center of the distribution coincides with 1, the true exposure value. Note that although the exposures are sufficiently well-defined for this search to be considered a success, there is still a substantial overlap between the estimated exposures of unexposed assets and those of exposed assets. In particular, since most assets with statistical exposures of 0.7 are actually unexposed, it is quite likely that the statistical factor model would recommend hedging one unexposed asset with another, or hedging an exposed asset with an unexposed asset.

The success criterion (1.2) does not quantify how successful a statistical factor model is. It only classifies a statistical search as successful or unsuccessful. To do better, define the statistical search *quality* as

$$Q_x^2 \equiv 1 - \frac{\langle \varepsilon^2 \rangle}{\langle (X + \varepsilon)^2 \rangle} = \frac{(X^T \hat{X})^2}{(X^T X)(\hat{X}^T \hat{X})}$$

The quality is the fraction of the statistically recovered exposures representing the true

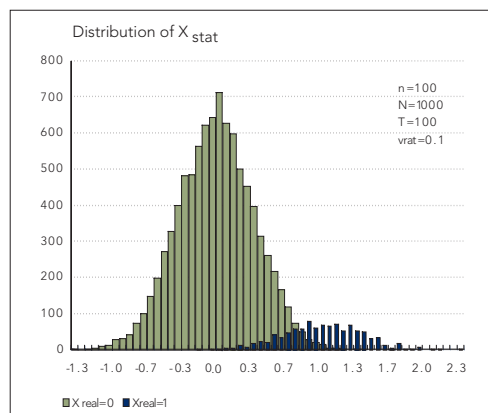


Figure 1

Histograms of statistically recovered exposures for unexposed (left) and exposed (right) assets. Ideally, exposed assets should all have exposures of 1, unexposed assets exposures of 0. Note that more unexposed assets have recovered exposures of 0.7 than exposed assets. Even with the extremely strong factor used in this simulation, hedging with the statistical exposures is unreliable.

exposures; the rest is error. Less technically, the quality can be thought of as the *overlap* between the true exposures and the exposures recovered through a statistical analysis.

Figures 2a–2c show how the quality of statistically recovered exposures varies with factor strength  $\sigma_f^2 / \sigma_s^2$ , exposed asset number  $n$ , and history length  $T$ , respectively. All of the searches involve a persistent factor ( $\tau/T=1$ ). Each figure plots lines for two families of searches: one within a smaller universe of  $N=100$  assets, and one within a larger universe of  $N=1000$  assets. The  $N=100$  case represents a search for a more broadly exposed factor, while the  $N=1000$  case represents a search for a more narrowly exposed “needle in a haystack”. Each figure depicts a set of variations on a benchmark statistical search conducted over  $T=100$  periods. In the benchmark case  $n=20$ , assets are exposed to an extremely strong factor with a variance ratio  $\sigma_f^2 / \sigma_s^2 = 0.1$ .

Figure 2a shows how exposure quality depends on factor strength. The scale of strengths ranges from negligible to unrealistically large. As mentioned earlier, a factor as strong as the market would typically have a strength or variance ratio of 0.3 or less. The strengths of hidden style factors or market-residual industrial factors would almost certainly lie below 0.05.

The figure confirms and quantifies two expectations. First, exposures to weaker factors are more difficult to recover accurately than those of stronger factors. Second, narrowly exposed factors of realistic strengths fiercely resist statistical factor searches. In the  $N=100$  case one asset in five is exposed, corresponding to a factor that works in a broad sector. In the  $N=1000$  case, one asset in fifty is exposed. Such a narrowly exposed factor could represent the distinction between biotech and traditional

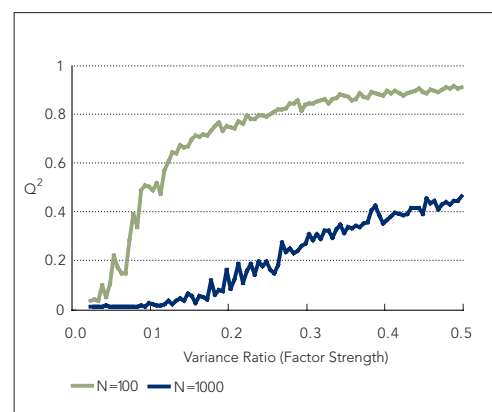


Figure 2a.

Quality of the statistically recovered exposures as a function of the strength of the hidden factor. The returns history consists of 100 periods. Twenty assets are exposed to the factor. The  $N=100$  line represents a factor search within a universe of 100 assets, the  $N=1000$  line to a “needle in a haystack” search within a universe of 1000 assets. In the latter case, the search is generally unsuccessful. Neither search is successful at factor strengths typical of strong styles or market-residual industries, which typically have variance ratios considerably less than 0.1.

pharmaceutical companies in a larger universe of stocks such as the S&P 1500.

Figure 2b explores in greater detail how the breadth or narrowness of a factor influences its statistical exposure quality. It confirms the hopelessness of finding and exploiting narrow factors with historical data of typical lengths. In particular, the statistical model is never successful when searching in the larger  $N=1000$  space, where the hidden factor only appears in one asset among ten or more.

If factor exposures persist unchanged, then given sufficient time a statistical search can identify the exposures to even a weak factor accurately. The problem is that for broadly exposed factors of reasonable strength, or even for very strong but narrowly exposed factors, a very long history of returns is required. Figure 2c shows how the history length affects

statistical factor quality. The scale extends to 100 periods - about two years of weekly data or half a year of daily data. The figure confirms the expectation that the accuracy of statistically recovered exposures increases with the usable history length. It demonstrates that the highest frequency data possible should be used in statistical factor searches. The data frequency is limited by the requirement that returns in successive periods must be statistically independent to an acceptable approximation. This suggests that returns periods as short as a day may be usable and desirable for liquid stocks, in particular for most high-cap and mid-cap stocks. Infrequently traded stocks may require longer periods.

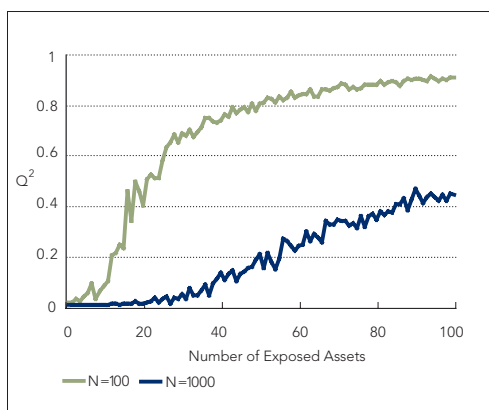


Figure 2b.

Quality of the statistically recovered factor exposures as a function of the number of assets that are exposed to the factor. Statistical factor models have trouble coping with even very strong factors that operate narrowly (affecting, for example, one asset in five).

The historical data requirements of statistical factor searches imply that, even with daily returns data, they will fail to capture dynamic factors for which the exposures change over the course of a year. An example of an important factor in this class would be a momentum factor that separates recent (half-year) winners from losers.

In summary, computer simulations support the picture of statistical factor searches painted by the success criterion (1.2). Although statistical models do not prescribe the details of the financial factors they seek, this very lack of specification limits their sensitivity and is likely to exclude many factors of interest to financial practitioners. These factors can be incorporated in a “well built” fundamental factor model. This observation leads to the next round of questions. Can statistical models find factors missing from good fundamental models? If so, how important are these factors?

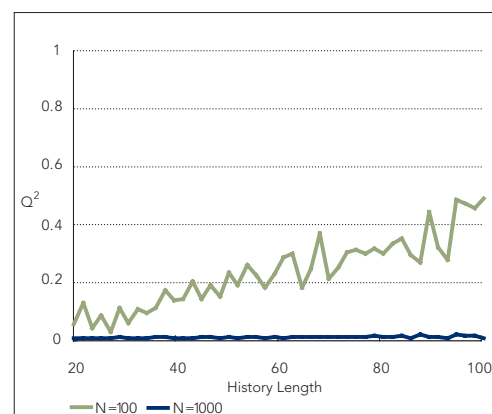


Figure 2c.

The dependence of statistically recovered factor exposure quality on the length of history applied to the statistical model. If a factor is sufficiently long-lived, a longer history will help define its behavior more clearly. Since the history length is measured by the number of statistically independent returns periods in the data, high frequency data are generally better than lower frequency data. In particular, daily data will be much more effective than monthly data.

### Implications for Risk Modeling

The preceding critique of statistical factor models teaches a valuable lesson. To build a good factor-based risk model, one should start by constructing the best possible fundamental factor model. Only fundamental factor modeling offers the opportunity to capture narrow industry distinctions and dynamic factors such

as momentum. To ensure that a strong, broad, and persistent factor has not been omitted, a statistical model can then be applied to the fundamental model's residual returns (the part of the returns not explained by the fundamental factors). To work well, the statistical model should use the highest frequency returns practical. Daily returns are attractive. Monthly frequencies are too low to be of practical use. The resulting model is a fundamental-statistical hybrid.

#### *Hybrid Factor Models*

The hybrid factor model retains many of the features of its fundamental component. Since the statistical factors in the hybrid model are based not on the asset returns themselves but on their returns residual to the fundamental model, the fundamental factor returns and exposures are unaffected by the presence of the statistical factors. The attribution of return to the fundamental factors therefore remains intact in the hybrid model. The hybrid inherits the all the interpretive power of the original fundamental model. On the other hand, risk attribution can become somewhat more complex, since the statistical factor returns can be correlated with the fundamental factors, and it may be impossible to separate the two sources of risk cleanly.

Since the statistical factors are expected to account for only a tiny fraction of asset-level return, it should be a safe approximation to ignore them in calculating the size of assets' specific returns and making specific risk forecasts. The specific risk forecasts of the fundamental model therefore suffice for the hybrid model as well.

By virtue of its secondary role in the hybrid scheme, the statistical model searches for factors which are likely to be marginal and obscure. In constructing the statistical compo-

nent of the hybrid model, it therefore is important to utilize the information in the residual asset returns as efficiently as possible. Several technical refinements to an ordinary principal components analysis aid in this. Large outlying returns that might mislead the statistical search can be identified and truncated to less extreme values. Standardizing the returns (dividing each asset's returns by the historical return standard deviation for that asset) can help the statistical factor search extract information more evenly from the entire field of data. In the applications described below, a similar procedure also was invoked along returns periods (instead of along assets) in the hope of evening the importance of different periods the data history - although in the end it seemed neither to improve nor to impair the search significantly.

#### **Hybridizing JPE3**

The discussion now turns to the problem of specification error in fundamental factor models. When a statistical factor model is applied to the residual returns of a fundamental model, what will it find? Can hybridizing a "well built" fundamental factor model with statistical factors improve the model's performance? How does hybridization affect the more delicate application of portfolio optimization?

#### *Hidden Factors and JPE3*

MSCI Barra's JPE3 model for Japanese equity risk is generally regarded as a soundly constructed fundamental factor model. It performs well in tests that review total and active risk forecasts for a multitude of portfolios. These include popular stock indices, portfolios constructed of either high- or low-capitalization stocks in individual industries, portfolios built from stocks with extreme value exposures, minimum risk portfolios, and portfolios containing random selections of stocks.

Most important, client experiences with the model have been very positive overall.

Despite its many successes, evidence for the existence of factors missing from JPE3 appeared in 2004, when optimization tests conducted in Barra's Tokyo office uncovered portfolios for which JPE3 significantly underforecasted risk. The portfolios were of two kinds, tracking portfolios and factor tilt portfolios. The tracking portfolios consisted of long-only positions held in a limited number of stocks (typically 75 or 200) selected from the JPE3 estimation universe. Subject to the long-only and asset number constraints, the tracking portfolios aimed to track the TOPIX index as closely as possible. The tilt portfolios also aimed for low tracking error against the TOPIX, but compromised this goal with attempts to improve expected return by tilting in favor of style factors such as Value.

In a sense, these optimized portfolios are unrealistic. The JPE3 estimation universe is very broad. It contains about 1,500 names, ranging from extremely large, liquid stocks to much smaller and less liquid ones. Seeking the best performance and with a broad field to select from, the optimizer tends to combine very large firms with much smaller ones in the portfolios uncritically. Actual portfolio managers are unlikely to take such positions. Also, portfolios freshly optimized every month without trading constraints incur enormous turnover, something a real portfolio manager would take pains to avoid. Nevertheless, there was no reason to expect that JPE3 would fail to produce good risk forecasts for these portfolios. When it did, research began to uncover the source of the problem.

It was apparent almost immediately that the inaccurate forecasts were due to "hidden factors", factors not included in the JPE3 model. The part of the risk forecast related to JPE3

factors worked very well. On the other hand, the specific returns attributed to the portfolios by the model (1) did not diversify properly and (2) sometimes were highly correlated with model factors such as Size. Both the failure to diversify and the correlations are clear evidence of hidden factors.

The first attempts to identify the missing factors were fundamental in nature. Although many trials were made, none of the proposed fundamental factors could fully account for the anomalous specific return behavior. Among the partial successes, one of the most effective efforts involved a Nonlinear Volatility factor; we will return to the implications of this below. Other explanations have been proposed - the "Daikou Henjou" migration of pension investments from corporate to government management for example, or options hedging around the Nikkei 225 - but no single one of them seems to fit all the facts. In the end, the failure of fundamentally motivated attempts to characterize the problem and fix the forecasts turned the effort to the construction of hybrid risk models.

The basic hybrid model deploys 3 statistical factors across the estimation universe. Risk estimates and exposures for the factors are based on a trailing 1-year window of daily data. In applications to the JPE3 problem the basic model equaled or outperformed variations that were intended to help with narrow factors. These variations included keeping the statistical factors confined to certain sectors, such as the financial or technology sectors, or applying separate sets of statistical factors to different ranges of capitalization. The results described below all refer to the basic hybrid model.

#### *Characteristics of the Hidden Factors*

The hybrid model's statistical factors successfully identified sources of returns commonality

hidden in the asset returns residual to the JPE3 model factors. Out-of-sample tests verified that statistically recovered patterns were real and not due to coincidences in the historical record.

The tests took statistical factor exposures derived from a year of trailing data and used them in regressions on the fundamental model's residual returns for the following month to estimate statistical factor returns (e.g., if the exposures reflected data up to and including May 2001, the regression calculated returns for the statistical factors in June 2001). The test then compared the size of the statistical factor returns to the size expected if the residual returns were truly asset-specific and there were no pattern to be found. The results consistently showed that the statistical exposures brought out nondiversifiable returns.

Table 1 displays the results of the out-of-sample testing in two separate historical periods, 1996/01-1999/12 and 2000/01-2004/06. It shows root-mean-square t-statistics for the monthly regressions - the average ratio of the statistical factor strength to the strength expected from diversified specific returns if no factor were actually present. Note that the strongest factor in-sample, "Factor 1", is not

| Period          | Factor 1 | Factor 2 | Factor 3 |
|-----------------|----------|----------|----------|
| 1996/01-1999/12 | 3.73     | 3.26     | 1.92     |
| 2000/01-2004/06 | 2.75     | 3.42     | 2.71     |
| 1996/01-2004/06 | 3.25     | 3.36     | 2.33     |

**Table 1.**

Significance of the statistical factors in out-of-sample regressions.

always strongest out of sample. The third strongest factor in-sample, "Factor 3", is always statistically significant, but is markedly weaker out-of-sample than the strongest out-of-sample factor.

Once it is established that the statistical factors capture real patterns in the historical data, questions related to the character of the hidden factors arise. How persistent are the hidden factor patterns? Do the exposures for the strongest in-sample factor in a benchmark month resemble the exposures of the statistical factors in other months? Note that the strongest statistical factor in one month may not be the strongest factor in another. It may even appear as an admixture of the other factors. To determine the persistence of the statistical factors, it therefore is best to compare the benchmark exposures against all of the statistical factors in the test month. The resemblance between factor exposures in different months is quantified by summing the overlaps between the strongest statistical factor in the benchmark month and all three statistical factors in the test month:

$$\text{Summed overlap} \equiv \sum_{j=1}^3 \frac{(X_{\text{benchmark},1}^t X_{\text{test},j}^t)^2}{X_{\text{benchmark},1}^t X_{\text{benchmark},1}^t X_{\text{test},j}^t X_{\text{test},j}^t}$$

(cf. equation (1.3) for the overlap between the statistically recovered exposures and the actual exposures).

Figure 3 shows the summed overlaps based on three benchmark months: January 1996, January 2000, and June 2004. The summed overlap is always perfect (equal to one) when the benchmark month and the test month coincide. Since a year of trailing data informs the statistical exposures, the benchmark exposures and the test exposures are only based on independent data when the benchmark and test months are separated by a year or more. When they are separated by less than a year, the exposures in both months are influenced

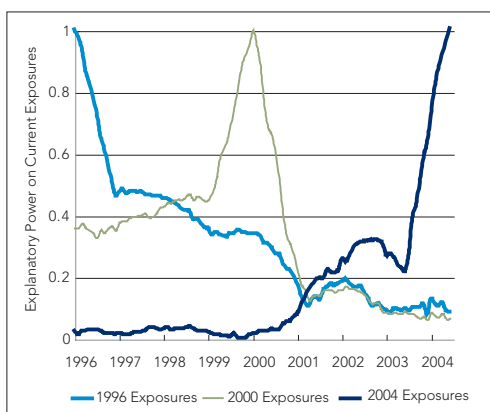


Figure 3.

Persistence of the statistical factors. The most important statistical factor in January 1996 remains closely related with the three most important statistical factors until 2000, when it loses relevance and is replaced by a weaker, unrelated factor.

by the same data (including the same coincidental patterns) and necessarily have high overlaps. Each summed overlap peak is thus about a year-wide. The “shoulders” that emerge from under each peak show the true level of the overlap. The figure shows that the strongest factor in 1996 perseveres through 2000 and then weakens significantly, almost vanishing from the post-2000 statistical factors. The statistical factors themselves seem to pass through a transition in 2000, when a new set of exposures becomes dominant (compare the overlaps for the 2000 benchmark and the 2004 benchmark). Thus the hidden factors appear to be transient, with lifetimes of several years. Their importance also varies over time. Note that the shoulders of the summed overlap with the June 2004 benchmark are considerably weaker than those associated with the January 2000 benchmark; this suggests that the leading statistical factor is weaker after 2001 than before. This is substantiated by Table 1, which shows that the leading statistical factor (“Factor 1”) was relatively weak during the 2000/01-2004/06 period.

The statistical factor exposures hint at the origin of the missing JPE3 factors: they have very significant correlations with the model exposures for Size, Trading Activity, and Volatility, as well as with the nonlinear combinations Size\_Size, Size\_(Trading Activity), and Size\_Volatility. (The nonlinear exposures are just products of the exposures for each of the constituent factors.) The correlations are significant both pre-2000 and post-2000, but are strongest in the earlier period. In fact, regressing the exposures for the strongest statistical factor in January 1999 against Size, Trading Activity, and their nonlinear counterparts yields an adjusted r-squared (or overlap) of 0.2. An overlap of 0.2 is quite large; Figure 3 suggests that the error content of the factor exposures is big enough that the maximum possible overlap of the statistical factors with the true explanatory factors is not too high, perhaps 0.7 or less. Thus, in agreement with the results of the earlier experiments with a nonlinear Volatility factor, it seems that at least some of the missing factor activity originates with transient stress between factor behaviors at low- and at high capitalizations. The JPE3 model does not adopt separate sets of factor returns in different capitalization ranges, and it favors high-capitalization assets in determining the factor returns. This “one size fits all” approach usually works well enough, but when investors treat the two capitalization ranges more distinctly than usual, hidden factors can appear and become particularly active among lower capitalization stocks.

#### *JPE3 and the Hybrid Model Compared*

With this evidence that the statistical factors are functioning properly, it is time to ask how hybridization influences the quality of risk forecasts. Do the forecasts improve, or do the statistical factors merely inflate them?

Furthermore, since portfolio optimization relies

on the effectiveness of the hedges recommended by a model, it places strenuous requirements on the quality of factor exposures. Are the statistical exposures good enough to improve the performance of optimized portfolios?

| Portfolio | Model  | Tracking Error | Bias |
|-----------|--------|----------------|------|
| Track 75  | JPE3   | 2.84%          | 1.36 |
| Track 75  | Hybrid | 2.59%          | 1.22 |
| Track 200 | JPE3   | 1.24%          | 1.50 |
| Track 200 | Hybrid | 1.18%          | 1.39 |

**Table 2.**

Tracking errors and bias statistics for tracking portfolios optimized under JPE3 and the JPE3-based hybrid model. The hybrid model reduces (but does not eliminate) the risk forecast biases. It also produces slight improvements in tracking.

Research on these questions is still in its early stages, but Table 2 offers some guidance. The table shows the results of optimization and risk forecasting together; i.e., the same model is used both to produce optimized tracking portfolios and to forecast the risk. This is intended to make for a fair comparison, since a portfolio optimized with a given model tends to nestle in a locus of underestimated risk. It would be less fair to optimize with JPE3, manufacture a portfolio for which JPE3 makes a particularly low risk forecast, and then let the hybrid model forecast its risk for comparison. The comparison would favor the hybrid model.

Table 2 uses the *bias statistic* to gauge the success of the risk forecasts. Technically, the bias statistic is the standard deviation of the forecast-normalized active return. (The active monthly return, the difference in return between the managed portfolio and its bench-

mark, is normalized by dividing it by the model's risk forecast.) More simply, the bias statistic can be thought of as the average ratio of realized to forecast risk. Accurate risk forecasts typically produce bias statistics  $b$  in the range,  $1 - \sqrt{2/T} < b < 1 + \sqrt{2/T}$ , where  $T$  is the number of returns periods in the study interval. Here the study covers the 5-year interval 1999/04-2004/03, so  $T=60$ . If the models are working well, their bias statistics thus should fall in the range  $0.82 < b < 1.18$ . Bias statistics below this range imply that the risk forecasts were too high, biases above the range that the forecasts were too low.

The bias statistics in Table 2 immediately make an important point: *all* of the forecasts, whether from JPE3 or the hybrid model, are biased. They are too low. On the other hand, the hybrid model does improve matters somewhat. It is a partial solution offering minor improvement, not a complete cure.

That hybridization does not merely inflate JPE3's risk forecasts is indicated by comparing the original and hybridized biases of forecasts that work well. An example is the risk forecast for the MSCI JP index, active against the TOPIX benchmark. Both models did very well in this case. Over the study period 1999/04-2004/03, JPE3 risk forecasts produced a bias statistic of 1.08. The bias statistic for the hybridized model was 1.05. Hybridization had little effect on this successful forecast.

To some extent, the bias results shown in Table 2 are unsurprising. It is unlikely that a fundamental model would omit a broad and exceedingly strong factor entirely. A strong missing factor is most likely to account for only about a percent or two of asset-level returns variance. Less and it would have little effect on all but the most highly diversified portfolios; more and it would not have been missed! Thus, even with a year of daily data the success criterion

(1.2) suggests that broad missing factors should be discernible but imperfectly recovered. This means the statistical factors will typically recover a fraction of the missing risk but not all or even most of it. To capture all of it, fundamental modeling based on a firm identification of the hidden factors would be necessary.

Because of estimation errors in the exposures to its statistical factors, the most difficult task for a hybrid model is to improve the performance of optimized portfolios. It is even possible that by suggesting ineffective hedges, the hybrid model might not do as well as the fundamental model it incorporates. Table 2 shows the performance of the optimized tracking portfolios, measured in terms of the tracking error achieved by each. The tracking error is the standard deviation of the active return. Tracking portfolios aim to minimize tracking error, so that they mimic the returns of the benchmark portfolio as accurately as possible. The differences between the tracking errors of the JPE3-optimized and hybrid-optimized portfolios are very slight and arguably insignificant. Still, it is impressive that the hybrid model is not inferior to the fundamental, and if anything, the hybrid surpasses the fundamental model very slightly. Perhaps enough of the exposure error has “averaged out” for a little of the information in the statistical factors to win through.

Given that the improvement in optimization may be quite small, the decision of which model to use for optimization must turn on other considerations, such as turnover and trading costs. Whether the changing statistical exposures of the hybrid model spur disproportionate turnover is difficult to tell in these portfolios. Since they are re-optimized every month, both JPE3 and the hybrid portfolios have very high turnovers - too high to make a good assessment of the statistical factors' impact. The fact that the statistical factors are

not much more dynamic than the fundamental model's existing technical factor exposures (e.g., Momentum, Volatility, and Trading Activity) and are likely to be less important than these factors suggests that there will be a noticeable but not overwhelming increase in the turnover of realistically managed portfolios. Until the nature of the compromise between performance and turnover becomes clearer, the best course may be to optimize with the fundamental model and forecast risk with the hybrid.

### Conclusions

Despite their claims to impartiality, the difficulties of undirected pattern searches make statistical factor models insensitive to factors that are narrow (i.e., that operate only over a fraction of the assets or returns periods in the search space), dynamic, or weak. Therefore, the best way to employ statistical factors seems to be to use them to scavenge for factors in the residual returns (the “leftovers”) of a well constructed fundamental model.

When statistical factors are used to extend a fundamental factor model, one should anticipate modest improvements in risk forecasting at best. The statistical factors are very unlikely to pick up all of the returns commonality that may have been overlooked in the fundamental model, and hence they cannot be expected to address all of the missing risk. The improvement in portfolio optimization should be even more slight. It remains unclear whether any performance gains purchased through optimizing with a hybrid model will outweigh the increase in turnover driven by the statistical factors. For the present, hybrid models offer an interesting extension to fundamental models for risk forecasting, but should be applied to optimization only with caution.

*Acknowledgement:*

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# Explaining Default Swap Variation

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## Introduction

Credit Default Swap (CDS) contracts have emerged as popular tools for hedging and trading credit risk. According to the International Swaps and Derivatives Association (ISDA) 2005 mid-year market survey, the notional amount of CDS contracts grew to \$ 12.43 trillion from \$5.44 trillion in mid 2004, representing a 128% increase. The number of corporate issuers whose obligations are reference entities for CDS contracts has grown to around 700. A natural outcome of this growth is the development of models and empirical studies to explain the levels and cross-sectional variation of credit spreads. This paper attempts to extend that effort.

A credit default swap is similar to an insurance contract. The buyer of insurance is protected against losses resulting from default by some issuer on a reference entity. In return for a premium known as the CDS spread, the seller of protection undertakes to make good losses upon default. In the most common case, the issuer is a corporation and the reference entity is a bond issued by the corporation. The premium is quoted on an annualized basis and is paid every quarter.

Intuitively, the variation in premium values across names indicates the relative likelihood of default by the issuers. This is more or less

correct if we assume the same recovery expectation across issuers. Thus, a crucial determinant of CDS spread level and variation is the probability of default by the issuer. Our goal here is to explore this relationship empirically.

## From Default Probability to Spreads

The CDS premium compensates the protection seller for the risk of loss due to default by the named issuer. To see this, consider a simple single-period model in which the premium and loss payments (if any) are made at the end of the period. Let  $p$  be the probability that the issuer of the reference entity defaults during the period of the contract,  $S$  be the CDS spread and  $R$  be the recovery in case of default.

The premium is computed by equating the present values of the premium payments and the expected payments by the protection seller. In our single-period model, this becomes

$$PV[S] = PV[p \times (1 - R) + (1 - p) \times 0],$$

where  $PV[.]$  denotes discounting to obtain the present value. We then have

$$S = p(1 - R). \quad (1.1)$$

The expression for the spread becomes more complicated as we take into account multiple premium payments and remove the artificial restriction that the loss payment be made at

the expiration of the original term of the contract; see [1] and [2] for details. However, the intuition that the CDS spread depends mainly on the default probability and the recovery rate remains the same.

We have implicitly assumed in the discussion so far that the default probability is appropriate for pricing credit-sensitive instruments. However, it is essential to keep in mind the distinction between a “pricing” default probability and a “real-world” (or physical) default probability. The real-world default probability is the likelihood that the firm will default due to the structure and inherent risk of its business and the economic environment. On the other hand, the pricing default probability is an implied probability, one that is consistent with market prices of instruments whose payoffs depend upon the firm's default. The pricing default probability implicitly accounts for the risk aversion of investors and includes compensation for investors willing to take on credit risk.

### Goals of This Study

What are the determinants of CDS spreads? In the previous section, we obtained the theoretical answer: default probabilities and recovery rates.

There are various methods of assessing default probabilities, and among these, quantitative models are gaining ground with practitioners in general and asset managers in particular. The Barra Default Probability (BDP) model is a quantitative approach [3] that forecasts the probability that a firm will default within a given time horizon. Ideally, such a model should be calibrated and tested thoroughly against actual firm defaults. This poses some problems, since defaults are relatively rare and model forecasts are numerous. To put this in perspective, we note that Moody's Investors Service has tracked about 1,500 default events in the last 35 years, whereas the BDP model makes about

10,000 forecasts of default probability every day.

Due to the sparseness of default data on the one hand and the desire to use default probabilities for the pricing of securities on the other, it is natural to investigate the relationship between model forecasts and market prices of credit sensitive instruments. In this study, we assess the effectiveness of the BDP model in explaining the cross-sectional variation of CDS spreads.

Default probabilities are dependent on firm variables such as leverage and business risk. Some studies have thus used firm level fundamental and equity market data as explanatory variables for CDS spreads; see for example, [4] and [5]. Such data are also inputs to the BDP model. One additional question we seek to answer in this study is whether the BDP is a better explanatory variable in a linear model than the collection of input variables typically used in structural models of default.

### Study

Our data consists of daily CDS spreads from Credit Market Analysis (CMA, on the web at [www.creditma.com](http://www.creditma.com)). CMA provides spreads in a hierarchical manner ranging from actual trades to values derived from a calibrated model. For our analysis, we use only actual trades or indicative quotes on five-year CDS contracts, since these are the most liquid contracts. We also require that a BDP value be available for each name included in our analysis.

We start our analysis with exploratory regression, using pooled CDS data for the month of April 2004. These data consist of 5,976 quotes or traded spreads on 450 distinct names. As mentioned earlier, we wish to determine if the BDP is more effective as an explanatory variable than the inputs used to compute it. The BDP model is similar to structural models of

credit risk in the sense that default is modeled in terms of the firm value, which in turn is inferred from equity market and leverage information. Specifically, the inputs to the BDP model are the firm's equity market capitalization, its equity volatility (forecast by a Barra equity risk model) and long and short-term debt information obtained from balance sheet data. For the purposes of comparison, we define a measure of leverage independent of the model-derived firm value. This measure, called *D\_TO\_A*, is a debt-to-assets ratio where the debt is the book value of total debt and the assets are computed as the sum of the book value of debt and the market value of equity. Thus, the set of variables being compared to the 5-year BDP (BDP5) are equity market cap (EQUITY), equity volatility (EQUITYVOL) and model-independent leverage (*D\_TO\_A*).

We measure the effectiveness of the BDP in explaining the cross-sectional variation of CDS spread levels with the adjusted R-squared metric for a series of regressions. The results are shown below:

| Number of Variables | Adjusted R-Squared | Variables                        |
|---------------------|--------------------|----------------------------------|
| 1                   | 0.44               | BDP5                             |
| 3                   | 0.31               | EQUITY, EQUITYVOL, <i>D_TO_A</i> |
| 2                   | 0.30               | EQUITY, EQUITYVOL                |
| 2                   | 0.29               | EQUITYVOL, <i>D_TO_A</i>         |
| 1                   | 0.28               | EQUITYVOL                        |
| 2                   | 0.06               | EQUITY, <i>D_TO_A</i>            |
| 1                   | 0.05               | EQUITY                           |
| 1                   | 0.01               | <i>D_TO_A</i>                    |

These results illustrate two important points. The equity volatility estimate is an important variable in that it explains more of the cross-sectional variation in CDS spreads than a combination of the equity and a leverage indicator. Furthermore, combinations of these input variables are inferior to the BDP alone in terms of explanatory power. This indicates that the BDP model captures effects not present in linear

functions of typical structural model inputs.

We conjecture that the adjusted R-squared obtained by using the BDP as an explanatory variable can be improved with the addition of other variables. This improvement in explanatory power might come from variations in CDS spreads based upon the domicile of the issuer of the reference entity and from market segmentation along investment grade and high yield lines. To test this conjecture, we add candidate variables for investment grade issuers and for the geographic region of the firm's domicile. Specifically, we estimate the following regression equation:

$$CDS_i = \beta_0 + \beta_1 \times BDP_i + \beta_2 \times IG + \beta_3 \times US + \beta_4 \times EU\_UK + \varepsilon_i \quad (1.2)$$

Here,  $CDS_i$  is the 5-year CDS, market premium (in basis points) on the  $i$ th firm and  $BDP_i$  is the forecast  $BDP_i$  (in basis points) for the 5-year horizon.

The CDS premium and BDP values are contemporaneous. The indicator variable *IG* is set to one for investment grade issuers and to zero otherwise. Similarly, the geographical region in which the firm is domiciled is also included in the regressions via the indicator variables *US* and *EU\_UK*.

The table below shows results from the cross-sectional regression for April 2004:

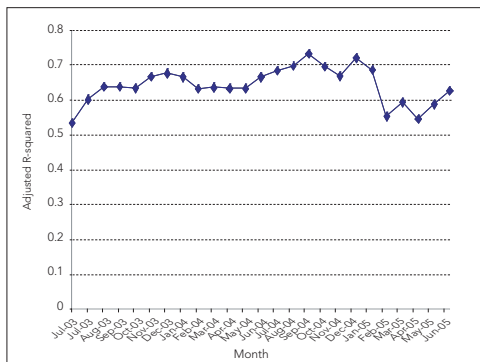
| Variable  | Parameter Estimate | Standard Error | t Statistic |
|-----------|--------------------|----------------|-------------|
| Intercept | 100.55             | 6.02           | 16.71       |
| BDP       | 0.0718             | 0.0037         | 19.44       |
| IG        | -121.98            | 3.59           | -34.01      |
| US        | 48.34              | 4.23           | 11.43       |
| EU_UK     | 28.68              | 4.26           | 6.73        |

We note that the high values of all t-statistics<sup>1</sup> indicate statistical significance at the 99.99% confidence level. The adjusted R-squared value for this multiple regression is 0.64. This is a significant increase from the value obtained when the BDP was the only independent variable.

<sup>1</sup> We compute robust standard errors to account for heteroskedasticity.

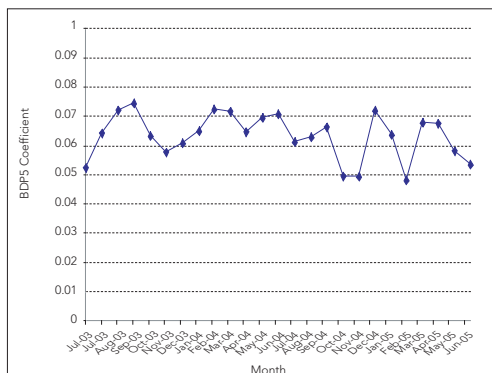
The results show that a substantial discount is available when the issuer is an investment grade firm and that a premium is charged for US issuers relative to EU/UK issuers. The discount on investment grade names may reflect a trend in CDS markets wherein there are far more contracts on investment grade names than on high-yield names. Indeed, in our data, the ratio of investment grade names to high-yield names is often roughly 3:1. The region premium could be due to the differences in CDS contracts for issuers in the two regions, generally pertaining to restructuring clauses.

In order to examine how the parameter values change with time, we run the regression for every month during the period July 2003 to July 2005. The adjusted R-squared value varies from month to month, but stays within a narrow range, as shown in the plot below:



The drop in the adjusted R-squared value in spring 2005 is related to the substantial widening of US automobile sector CDS spreads coincident with ratings downgrades of some firms.

The estimated value of the slope parameter for the BDP value also varies during this period, as shown in the plot below:



The relationship in equation (1.2) can be subjected to a stricter test by using daily differences in the levels of the CDS premium as the dependent variable and the daily change in the BDP as the explanatory variable. We thus formulate a new regression model as follows:

$$CDS_t^\Delta = \beta_0^\Delta + \beta_1^\Delta \times BDP_t^\Delta + \epsilon_t^\Delta \quad (1.3)$$

Here, the superscripts serve to remind us that the regression is for differences in variables and that the parameters are different from the ones in equation (1.2). The indicator variables in equation (1.2) are excluded since they do not change from day to day.

The results from this single variable regression are as follows:

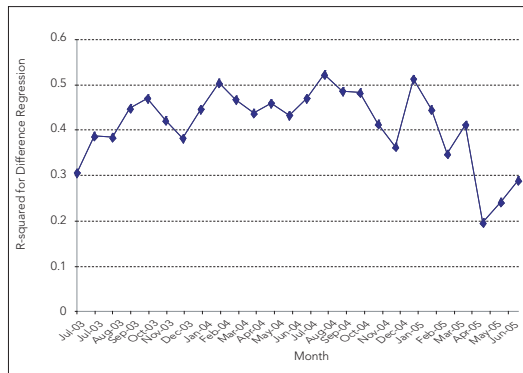
| Variable     | Parameter Estimate | Standard Error | t Statistic |
|--------------|--------------------|----------------|-------------|
| Intercept    | -0.0005            | 0.36           | -0.00       |
| $BDP^\Delta$ | 0.103              | 0.01           | 10.11       |

The R-squared value from this regression is 0.44. The lowered R-squared value relative to the levels regression indicates that the BDP is a more reactive measure of creditworthiness than the CDS spread. This is consistent with it being strongly dependent on equity market inputs, which tend to fluctuate more than CDS spreads. The parameter estimate for  $BDP^\Delta$  is similar in magnitude to that obtained in the multiple regression for CDS levels, giving us increased confidence in the relationship between CDS spreads and BDP values.

The study in [5] uses similar procedures, but considers leverage, equity volatility and risk-free rates as explanatory variables. The R-squared values with those variables are reported in [5] to be approximately 0.60 and 0.23 for level and difference regressions respectively.

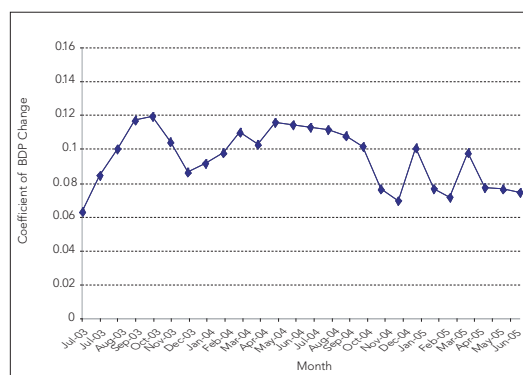
With a series of monthly regressions using pooled data, we can examine how the

explanatory power of the differences regression in equation (1.3) has varied with time:



Again, we see a pronounced drop in the R-squared value in spring 2005, coincident with the auto sector widening in the CDS markets.

The slope coefficient of  $BDP^A$  in equation (1.3) varies with time in a manner roughly parallel to that of the coefficient in the levels regression:



### The Credit Risk Premium

As described in Section 2, the CDS premium is a product of the *pricing* default probability and the loss given default. On the other hand, if a real-world default probability is known, the expected loss would be given by

$$E[\text{loss}] = (1 - R) P_{RW}$$

where  $P_{RW}$  is the real-world default probability and  $R$  is the expected recovery rate. The risk premium would then be the difference between the CDS premium and the expected loss. The slope coefficient of regressions of CDS spreads against default probability measures can be used to draw conclusions about the credit risk premium. This is pursued, for example, in [6] and [7].

Measurement of the credit risk premium relies on having good estimates of a real-world default probability and expected recovery rates. In order to establish the usefulness of the BDP model in forecasting real-world defaults, we tested it against historical default experience. We find that the model shows good default discriminatory power relative to agency ratings [3]. Good default discrimination reflects the ability of the model to rank firms as being more or less creditworthy relative to other firms. Any rank-preserving transformation of the BDP values would thus produce the same forecasting power. As a result, these tests do not allow us to determine the actual level of the real-world probability of default.

Secondly, *ex ante* expected recovery rates are not reliably represented by averages. Studies of historical rates of recovery demonstrate that there are substantial variations in the recovery rate, both across industries and time, see for example, [8] and [9].

We continue this line of empirical research to surmount these difficulties, which are inherent to some extent in all studies of the credit risk premium and its time variation.

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# A Prepayment Model for the Danish MBB Market

Oren Cheyette

## Introduction

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The Danish market for mortgage backed bonds (MBB) is one of the oldest and largest in the world. It can look back on a history of over 300 years and its only rival in size in Europe is the German Pfandbrief market. Danish mortgage bonds are part of the Lehman Global Aggregate Bond Index and attract an increasing amount of interest from foreign investors.

Mortgage backed bonds differ in various aspects from garden-variety bonds: in addition to interest rate risk they are exposed to both borrower and bank credit risk, and to mortgage-specific prepayment risk, "i.e." the fact that some mortgagors prepay the principal of their loans ahead of schedule when it becomes financially advantageous for them to do so. The credit risk of Danish MBB is considered to be small<sup>1</sup>, but the cost to investors from potential prepayments cannot be neglected either in valuing the bonds or in calculating their exposures to market risk factors.

In this article, we introduce a variation of the Implied Prepayment model to calculate

spread values and effective Durations of Danish MBB. We start with a brief introduction to the Danish MBB market and mortgage bond series. In the following sections we then point out some significant differences between the Danish and the US markets and discuss methods of prepayment modeling. We conclude with a practical application of the theories discussed in this article and present our results for credit spreads and effective durations.

## Market Overview

Mortgage banks in Denmark are specialized institutions committed by law to mortgage financing only. Currently, there are 8 Danish mortgage banks: BRFKredit, DLR Kredit, FIH Realkredit, LR Kredit, Nordea Kredit, Nykredit, Realkredit Danmark and Totalkredit. Totalkredit used to be an independent entity but is now a fully owned subsidiary of Nykredit. The five largest banks - BRFKredit, Nordea Realkredit, Nykredit/Totalkredit and Realkredit Danmark - together account for roughly 95% of all issued Danish mortgage bonds.

<sup>1</sup> Based on actual payments missed, non-performing loans are around 0.1% of total loans.

The issuance of mortgage bonds is regulated by the Mortgage Credit Act which was first introduced into Danish legislation in 1848 and remains almost unchanged to the present day. The foundation of this legislation is the so-called Balance Principle which regulates the financial risk of the mortgage credit institution resulting from differences in payments between loans and funding, fluctuations in interest and exchange rates and the use of financial instruments.

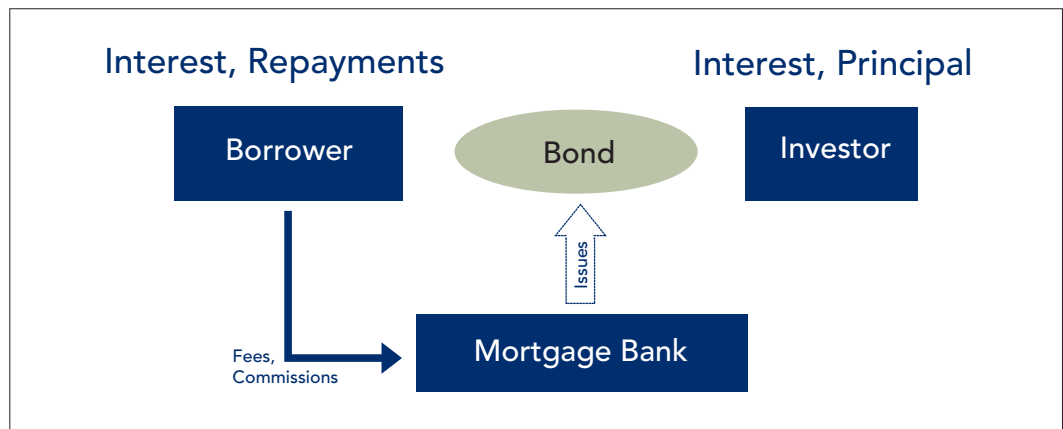
The effect of the Balance Principle is that Danish mortgage banks must fund all their lending activity by issuing mortgage bonds, and that the principal and interest cash flows from mortgage borrowers must match those

paid to investors very closely<sup>2</sup>. Regulations tightly restrict the asset/liability mismatch of the lenders; therefore mortgage bonds effectively function as pass-throughs of the underlying home loans (see Figure 1).

In addition there exist further legislation which enhance the quality of Danish mortgage bonds (limits on LTV ratios for mortgage loans, valuation rules for mortgaged properties) as well as rules to protect investors in case of non-performing loans (required cash reserve levels for mortgage banks, bankruptcy remoteness from the mortgage credit institution). Altogether this regulatory framework results in a very healthy market with extremely low delinquency rates.

Figure 1

The Balance Principle in action - equal cash flows from the borrowers to the investors with the mortgage bonds acting as pass-throughs.



### Danish Mortgage Backed Bond Series

Since the Danish mortgage market is highly standardized, the offered bond series may be presumed to be homogeneous in exposures to credit and market risk across issuers. This view is supported by a common market-making scheme, created by the individual Danish mortgage banks to maintain and support the secondary market. As a result, the market for Danish mortgage bonds is very liquid.

Due to the Balance Principle mortgage bonds generally mirror the terms of the underlying mortgage loans with regards to the nominal amount, coupon and other terms. The bonds are issued in individual 'series' on tap by the mortgage banks; a standard 30-year callable mortgage bond is usually open for issuance for three years. Each bond series increases in size as more and more mortgage loans are granted and a corresponding equal amount of bonds are issued<sup>3</sup>.

<sup>2</sup> In this context, 'very closely' means that Danish mortgage bonds are usually backed by mortgages with the exact same coupon rate and payment schedule.

<sup>3</sup> This method of issuing new bonds when needed is known as a 'matching tap issue'.

Most mortgagors in Denmark either choose long-term fixed rate loans or yearly adjustable loans and it therefore comes as no surprise that the most common mortgage bonds issued in Denmark are:

1. Callable annuity bonds with quarterly interest payments
2. Non-callable bullet bonds with a maturity of  $n$  years and annual interest payments.

These bonds provide the funding for the majority of Danish mortgages although there are exceptions: the Danish legislature has adopted new regulations in recent years which made it possible for mortgage lenders to also offer loans with interest-only periods and capped variable interest loans, but these account for only a fraction of all mortgages (see Table 1 below).

|                  | Amount (billion euro) |       | Market Share in percent |      |
|------------------|-----------------------|-------|-------------------------|------|
|                  | 2003                  | 2004  | 2003                    | 2004 |
| Bullet Bonds     | 96.9                  | 125.4 | 43.7                    | 51.8 |
| Fixed Rate Bonds | 116.0                 | 109.0 | 52.3                    | 45.0 |
| Other Bonds      | 8.7                   | 8.0   | 3.9                     | 3.3  |

Due to the Balance Principle each adjustable mortgage (i.e. ARM or interest-only) has to be completely refinanced with a new bond issue at the end of each period. Examples are:

- A ten-year ARM with an interest-reset period every year would involve 10 different bond issues of one-year bonds, each one financing the remaining balance of the loan.
- A thirty-year fixed rate loan with an interest-only period of 5 years would involve one bond series for the interest-only loan and a new issue of a 25-year fixed rate bond after 5 years covering the remaining balance (i.e. the original loan amount in the case of no prepayment)

### Denmark is Different

We assume that most readers will be more familiar with American mortgage backed securities (MBS) issues than with Danish mortgage backed bonds. It is therefore necessary - especially with regards to later developing a risk model for MBB's - to point out some of the

idiosyncrasies of Danish MBB.

One big difference is the general assumability of Danish mortgage loans. Contrary to the situation in the US it is therefore often not necessary to finance the purchase of an existing house with a new mortgage if the new owner chooses to assume the mortgage of the previous owner. With respect to prepayment behavior we expect that there will not be a seasonal dependence of mortgage activity in Denmark, as it is observed in the US.

The Danish mortgage system also makes it easy for borrowers to refinance their mortgages. Fixed rate loans are usually callable so that the borrower may redeem his mortgage by repaying the loan at par. This prepayment option is valuable to the borrower and is priced into the market price of the mortgage. In addition, Danish borrowers also can choose to use the so-called 'delivery option', i.e. purchasing the bonds of the same type that the lender sold to fund the loan in question, and handing them over to the mortgage bank. This option is available inde-

pendently of whether there is an underlying call option for the loan or not.

Due to the Balance Principle, any repayments on the asset side have to be matched by a reduction of bonds in circulation on the liability side. Mortgagors have to announce their wish to prepay part of their loan in advance (the exact time period depends on the frequency of the repayment schedule - usually two months in case of a quarterly schedule). Every lender then compiles the prepayment data for a given coupon payment date and publishes it on a weekly basis. The amount of MBBs to be called is then fixed at some point before the coupon payment date (usually two weeks) and the retirement of the MBBs itself is done at the same time as the coupon payment. It should be pointed out that whereas usage of the call option does count as a prepayment - i.e. bond investors have part of their holdings involuntarily called - usage of the delivery option does not. This is due to the fact that the bonds used in the case of the delivery option are already in the hands of the mortgagor and the offset of the loan against the bond delivery does not affect the other bond holders.

The existence of the call and delivery options encourages mortgage borrowers to undertake some rather fancy refinancing activity. As in the US market, it is usual to see a rise in refinancing during falling interest rate periods when mortgagors replace high interest rate loans with new lower interest rate loans. But in addition to that, Danish mortgagors are also known to refinance in periods of rising interest rates since in this case the old mortgage bonds can be purchased below par on the open market. Using the delivery option therefore results in replacing a low interest rate loan with a loan of higher interest rates, but smaller size. This can be advantageous due to tax reasons, and of course the high interest rate loan can be refinanced again should the interest rates drop back to normal levels.

### Prepayment Modeling

As mentioned before, the main sources of risk to investors in Danish MBB are interest rate risk and spread risk. The latter is primarily driven by uncertainty around prepayments, since the amount of default risk is negligible. Interest rate risk is defined as the price uncertainty of MBB's with regards to unknown future interest rates. Changes in the interest rate affect MBB prices by changing the discount factors that have to be applied in the evaluation of current MBB prices, as is the case with other fixed income securities. In addition, due to the fact that mortgage current coupons tend to be correlated with long-term interest rates - and since mortgagors tend to prepay when current coupons sink below their old mortgage loans we can also observe that changes in the interest rate environment affect prepayment speeds and thereby also MBB prices.

For MBB's we therefore obtain a price dependence on interest rate changes which is stronger than what we would expect from comparable plain vanilla bonds. We model the uncertainty of future interest rates using an option theoretic approach, "i.e." by generating a probabilistic simulation of future interest rate scenarios and discounting the expected cashflows - under consideration of prepayment activity - to estimate current market prices. The exposure of MBB's to interest rate movements is then calculated by deliberately changing the initial term structure curve and observing the resulting percentage price sensitivity of the valuation model.

We value mortgages by generating a set of interest rate scenarios based on the Hull-White model of interest rate dynamics. Based on these scenarios we project future MBB cashflows and then compute the present value of an average of these cashflows. In this way we capture the impact of uncertain future mortgage cashflows due to future interest rate variability on the investor.

Focusing on prepayment risk now, we should first mention that this includes uncertainties in prepayment speeds beyond the already mentioned interest rate effects - an example would be the risk that mortgagors do not use their prepayment option optimally. Exposures to the prepayment factor are determined in the same manner as for interest rate exposures: by shocking the overall speed of the prepayment model and observing the resulting changes in the valuation model.

Before we continue in our discussion about prepayment models, let us first introduce a few concepts. The principal payment  $p$  of a mortgage can be decomposed into two parts: a scheduled part as per the Terms and Conditions of the mortgage and a prepayment part:

$$p_{\text{Principal}} = p_{\text{Scheduled}} + p_{\text{Prepay}}$$

These prepayments are described using the single monthly mortality rate (SMM), defined as:

$$p_{\text{Prepay}} = \text{SMM} \cdot (b - p_{\text{Scheduled}})$$

Where  $b$  equals the remaining balance at the beginning of the month. The SMM ranges between 0.0 and 1.0; if it equals 0.0 then no principal is being prepaid, if it equals 1.0 then the whole remaining balance is being paid in full. The question for the practitioner is then: how do we determine and predict the single monthly mortality rate?

#### *Prepayment Models I: Econometric Models*

The most common practice is to build a behavioral model of when mortgagors prepay their mortgages - we will call such a model econometric since it relies solely on economic input data (e.g. interest rates, age of the mortgage in question, etc). Starting from general considerations one can then derive a SMM, adjusting the intricacy

of the function to the desired degree.

An example for this kind of model is the prepayment model Barra is currently using for the US market.

The second step in building an econometric model is then to fit the model parameters using several years of prepayment data, preferably during times of varying interest rate environments and using mortgage series with different characteristics. The resulting function can then be used to predict prepayment rates in future interest rate scenarios, but this already hints at one of the problems which *burdens* econometric models: since they use vast amounts of historical data, by their construction they are unsuited to quickly adapt to changes in market structure or sentiment.

Another drawback is the fact that econometric models usually fail to calculate spreads in a consistent manner. Considering our own Barra US Prepayment Model, we sometimes observe a substantial deviation across MBS coupon classes in the option adjusted spreads (OAS) - clearly an undesirable artifact of the model since the spread risk should not depend on the coupon of the underlying mortgage once the prepayments have been taken into account.

#### *Prepayment Models II: Implied Models*

This brings us to the second class of prepayment models: the *implied* prepayment models<sup>4</sup>, so called because they imply future prepayments from current market prices. It is therefore not so much a prepayment model as rather a valuation model. This approach has the benefit that not only is the market expectation of prepayment rates of MBS included in the market price, but also the price for prepayment risk. Contrary to the econometric models, which evaluate MBS in a risk-neutral framework, an

<sup>4</sup> For further reference: Cheyette, Oren "Implied Prepayments", Journal of Portfolio Management (Fall 1996).

implied model therefore takes into account the changing market price of risk for prepayment uncertainty.

The practical approach in an implied model is to fit a prepayment model to market prices<sup>5</sup> and treat MBS spreads as an additional model parameter - not as an output, as is the case in econometric models - and in such a way get consistent OAS by construction. In doing so one can completely separate interest rate risk from prepayment risk and the spread is truly a measure of residual risk other than that due to model uncertainty or the market price of prepayment risk. Apart from producing consistent OAS this approach also has the added advantage of being far less data intensive than the econometric approach, since all that is needed are current market prices.

It should be noted, however, that the prepayment rates calculated within an implied model are not as easily interpreted as those in an econometric model. Since changes in the model prepayment speeds in an implied model might be caused by either changes in the prepayment rates or changes in the market price of prepayment risk, one can not draw immediate conclusions about corresponding changes in mortgagor behavior. For the same reason, implied models also can not be used to forecast future cashflows. Using an econometric model in a risk neutral valuation setting identifies both model error and the market price of prepayment risk with variation of spread across coupons (and over time). It does, however permit cashflow forecasting, which the implied model does not.

To summarize our comparison between these two model approaches, the econometric modeling approach is advantageous if one needs accurate predictions of future cashflows, but of limited use

for valuations. However, if valuation - and risk estimation - is the main concern for investors, implied prepayment models should instead be used for these purposes since they provide a detailed picture of the discounted present values adjusted for the market price of risk.

### Implementation and Estimation Universe

Due to the aforementioned features of the Danish mortgage market we assume that the single monthly mortality rate is dominated by mortgagors refinancing their loan to capture more favorable interest rates. This leads us to consider the following functional form:

$$SMM = \alpha \cdot \left[ \log \left( 1 + \exp \left( \gamma \cdot (r - r_0 - \beta) \right) \right) + \delta \right]$$

Here,  $r - r_0$  is the difference between the current coupon rate  $r_0$  of the existing mortgage and the available interest rate for refinancing, in our model estimated to be the 10 year spot rate. The remaining parameters serve as fitting parameters and are determined using a non-linear fit algorithm which minimizes sum of squared pricing errors. The fitting parameters can be identified as follows: overall scale parameter  $\alpha$ , refinancing incentive offset  $\beta$ , refinancing rate dependence  $\gamma$  and refinancing baseline activity  $\delta$ .

Using current market prices as input parameters for our fitting algorithm we determine model prices and spreads for the mortgage bonds. Since we calculate spreads and prepayment speeds in the same process, our Danish MBB model combines the - usually separate - steps of first calculating a risk-neutral prepayment model and then adding an implied prepayment model to take into account the changes in prepayment risk. It is therefore necessary to stress the point that our constant prepayment rate (CPR) predictions are not the commonly used econometric CPR values found in other publications, but

<sup>5</sup> This prepayment model might actually be an econometric model - the result would be a 'best of two worlds' model combining the advantages of both approaches.

instead are calculated within a framework that incorporates both prepayment forecasts and risk pricing for uncertainty in those forecasts.

We use the following criteria to select the Danish MBB which we include in our estimation universe:

- Fixed-rate annuities, maturities from 10 to 30 years
- The open period has ended and there are no more tap issues forthcoming
- The outstanding amount is greater than €100 Million
- Available data from our vendors with reliable series information - this restricts our universe to issues from Nykredit, BRF Kredit and Realkredit Danmark

Using these selection criteria we end up with an estimation universe of roughly 40 MBBs<sup>6</sup> at any time with

- 3%—7% coupon rates
- 10—30 year maturities
- Roughly €50 Billion combined amount outstanding

We proceed to calculate model prices for this subset of MBB within our framework, using the SMM function as stated above. In an iterative process we then determine the optimal combination of OAS level and parameter values for our SMM by minimizing pricing errors. The SMM parameters and the spread level have to be re-estimated every month to allow for changes in prepayment behavior and risk pricing. We observe changes in the model parameters on a month-to-month basis, which is to be expected since especially the price for risk can be quite volatile.

### Results

To show the change in our model parameters within a six-month time frame, we present parameter estimates and derived values for Danish MBBs within the estimation universe for two months: 04/2005 and 11/2005. The parameters for the functional form of the single monthly mortality rate as given in the equation above are given in the table below:

Table 2

|               | OAS | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---------------|-----|----------|---------|----------|----------|
| April 2005    | 67  | 0.0087   | 0       | 2.4      | -0.02    |
| November 2005 | 105 | 0.0056   | 0       | 0.9      | 1.2      |

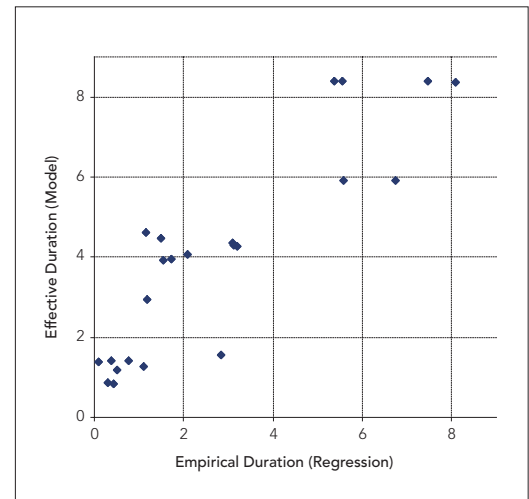
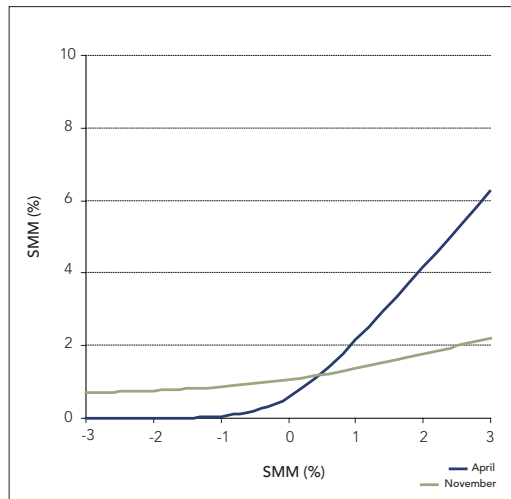
As mentioned above, we generate one common OAS for all MBS issues, given in the first column. It is noticeable that the  $\beta$  parameter is zero in both cases, which is the result of a constraint we impose. We introduced the constraint  $\beta > 0$  for two reasons: to improve the behavior of the numerical fitting algorithm and to restrict borrowers to only prepay when the coupon rate is at least as low as the coupon of their existing mortgage.

The prepayment functions for these two months take the following form with :

As expected, we see a higher SMM with increasing refinancing incentive  $\chi$ . The flattening in the SMM curve we observe in the time period from April to November could either hint at a change in mortgagors' prepayment behavior or at a shift in the market price for prepayment risk.

<sup>6</sup> Out of approximately 600 different series.

Figures 2 and 3



To get a feeling for the quality of our model we did a short exercise to validate the effective durations calculated using our approach. The idea is as follows: starting from a Danish government bond (i.e. bearing neither credit nor prepayment risk) we obtain a time series of bond returns (we denote MBB returns as  $r_{MBB}$  and government treasury returns as  $r_T$ ) and durations ( $D_{MBB}$  and  $D_T$ ). We then run a regression between the Danish government bond and our Danish MBB return data to determine  $\beta$ , defined as:

$$\beta = \frac{\text{cov}(r_{MBB}, r_T)}{\text{var}(r_T)}$$

In a rough approximation we can then calculate 'empirical' bond durations as:  $D_{MBB} = \beta \cdot D_T$

As can be seen in the plot above, the effective bond durations, calculated with our implied prepayment model, are in reasonably good agreement with the empirical bond durations, estimated using the regression (data as of 11/2005).

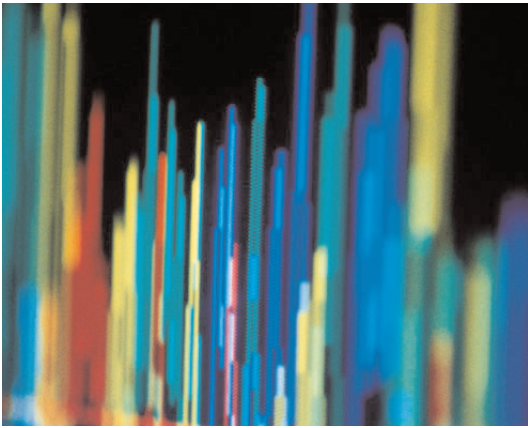
To get a quantitative feeling for the quality of the duration relationship we performed a cross-sectional regression between the empirical and implied durations. We set the intercept to be zero and got a linear fit with  $D_{MBB} = 1.24 \cdot D_T$

and an adjusted  $R^2 = 0.85$ .

### Summary and Outlook

The calculation of Danish MBB value and risk, using a traditional option theoretic approach, encounters the problem of applying risk neutral valuation methods to the uncertain cashflow predictions of an econometric prepayment model. A consistent treatment of MBB valuation can not use a risk neutral framework, but instead must account for the market price of prepayment risk, since prepayment uncertainty affects all MBB and investors require compensation for bearing this additional risk.

By using an implied prepayment model, constructed by fitting a generic functional form to market prices of liquid Danish MBB, we take the market price of prepayment risk into account and produce consistent results. Since we only use current pricing data as a model input this approach does not require access to a historical database of prepayment data. This allows us to extend our model to cover different markets with comparative ease. A brief foray into Japanese MBS shows promising first results, and we look forward to present them in more detail in a future publication. ■



# Dynamic Volatility

## and its Implications for Portfolio Management

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In the Autumn 2004 Horizon Newsletter, we looked at the decline in cross-sectional volatility in the US market that followed the burst of the technology bubble, and examined its sources. In this report, we extend the study to other developed markets and simulate the impact of dynamically changing volatility on passive and active investment strategies.

### Highlights

- We have observed a significant decline in cross-sectional volatility - defined as the dispersion of asset returns at a single point in time - in major equity markets since its peak during the bubble.
- Analyzing the common-factor portion of cross-sectional volatility, we find that in the US and European equity markets the relative importance of industry and style return dispersion has shifted significantly. Industries, after a relative decline in importance during the bubble years, increasingly dominate styles over the last three years.
- In Japan, style factors such as Volatility and Momentum play a significantly larger role than in the US and Europe, implying that

active managers need to be more conscious about their exposures to these factors.

- The level of cross-sectional volatility has significant implications for active and passive investment strategies. The optimal level of tracking error, the size of active exposures and the optimal number of securities may vary wildly with changing volatility levels. Investment managers should analyze the sources of changing volatility levels and the potential impact on their expected returns before adjusting their strategies.

### Introduction

The dispersion of returns across assets at a single point in time-cross-sectional volatility (CSV) is a good gauge of the opportunities for generating active returns through security selection<sup>1</sup>. If all assets had the same return, CSV would be zero. The opportunity to generate active returns scales with the level of CSV. In times of high CSV, a portfolio's tracking error (the risk of deviating from its benchmark) tends to increase as well. This in turn has direct implications for portfolio management mandates with tracking-error

<sup>1</sup> See E. Senechal, 'The Challenges of Declining Cross-Sectional Volatility', Horizon Newsletter, Autumn 2004, for a thorough discussion of the sources of cross-sectional volatility in the US Market, [http://barra.com/news/pdfs/Horizon178\\_Autumn2004.pdf](http://barra.com/news/pdfs/Horizon178_Autumn2004.pdf).

targets. Keeping tracking error stable when CSV levels change significantly, requires considerable adjustments to the portfolio's composition.

We define CSV as the standard deviation of monthly stock returns. Each stock's return represents one observation in the computation of the standard deviation for a given month and each observation is weighted by the market capitalization of the stock. Capitalization-weighted CSV provides a more accurate picture of the *investable* opportunities created by the dispersion of asset returns compared to equal-weighted CSV. We include all assets in Barra's model estimation universe for the relevant markets when we compute these volatilities.

Cross-sectional return dispersion can be divided into asset-specific return dispersion and dispersion driven by common factors—the categories used by factor models to explain the risk and return of single securities or portfolios<sup>2</sup>. The decomposition into asset-specific and common-factor dispersion can help us understand the type of active investment strategies that are more likely to be affected by changes in CSV. For example, in an environment in which common factor return dispersion has increased in

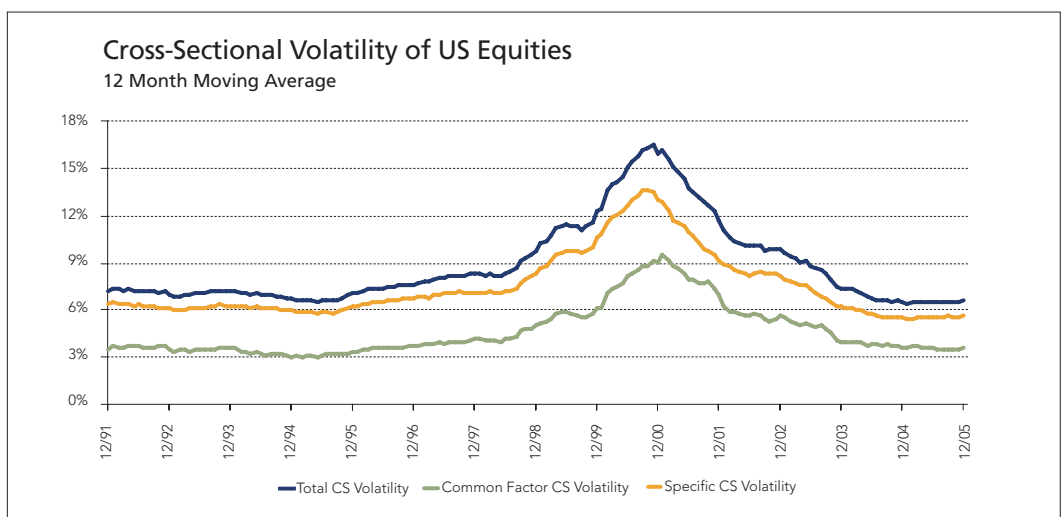
importance relative to specific returns, managers who incorporate style and industry bets in their portfolio strategies will find *relatively* more opportunities to add value.

In this report, we first analyze the dynamics of CSV in large developed equity markets for periods ending in December 2005. We then simulate the effect of dynamically changing volatility levels on different investment strategies. We simulate passive and active strategies in the US, Japanese and UK markets. The passive strategies track market indices with a limited number of securities, and the active strategies simulate value managers with fixed tracking error targets. Changes in volatility levels have a direct impact on both types of strategies with regards to optimal portfolio construction and performance expectations.

### *The US Equity Market, 1992 to 2005*

Between 2001 and 2003, after the bursting of the internet bubble, the CSV in the US market dropped rapidly (see Figure 1). CSV has recently stabilized at relatively low levels last seen in the early to mid 1990s. Because it is important to understand the sources of CSV, we look not only at the total CSV but also at its components, common-factor and asset-specific CSV.

Figure 1



<sup>2</sup> Single security as well as Index returns can be explained by a combination of so called common factors and asset specific returns. MSCI Barra's equity model common factors are Industries and risk indices and influence all stocks within a given market. Risk indices are a combination of fundamental and market driven data that are relevant to explain portfolio returns and risk patterns. Risk factors for the MSCI Barra Equity models include fundamental data like historical and forecasted earnings, book value, financial leverage, dividends, as well as price driven data like the momentum or relative strength, the recent volatility in stock prices, the market capitalization of stocks, etc.

The percentage of common-factor CSV is defined as the ratio of CSV of stock returns explained by the common factors to the total CSV of stock returns. This ratio tells us what proportion of asset return variation is explained by market-wide or industry-wide events as opposed to firm-specific events.

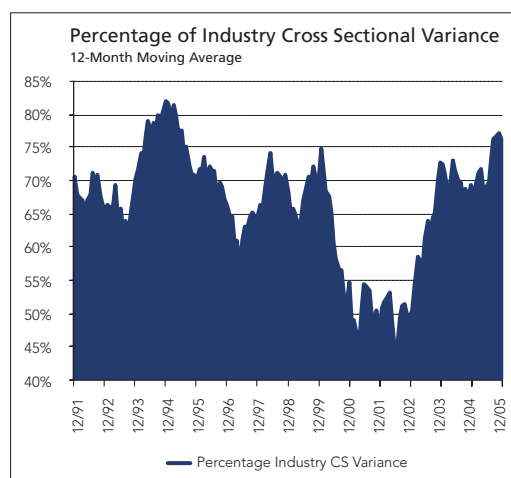
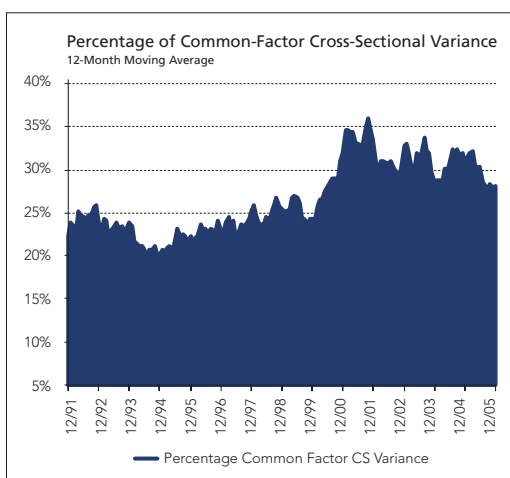
As seen in Figure 2, there has been a significant increase in the share of CSV due to common factors since the late 1990s, though it seems to have stabilized over the last few years. These higher levels indicate that managers who use strategies based on sector- or

style-rotation may have found relatively more opportunities to generate active returns than in the mid 1990s.

To gain further insight into the sources of common-factor CSV, we look at the relative importance of industries versus styles.

We define the percentage of industry CSV as the ratio of CSV of stock returns due to industry factors to the total CSV due to common factors. This tells us how much asset-return variations due to common factors are explained by industries as opposed to styles.

Figures 2 and 3



Although the level of common-factor CSV has been relatively constant over the last few years, its composition has changed dramatically. Figure 3 shows that style factors had become almost as important as industry factors in 2000 and 2001. This dramatic increase in style factor-related return dispersion is largely attributable to three factors in Barra's US Equity model: Momentum (a measure of relative strength), Volatility and, to a lesser extent, Earnings Yield (which consists of different earnings-to-price ratios). These three factors saw very high negative or positive returns towards the end of the technology bubble and during its subsequent collapse.

The share of common-factor CSV due to style factors has since fallen back to levels seen in 1994 and 1995—less than 25%.

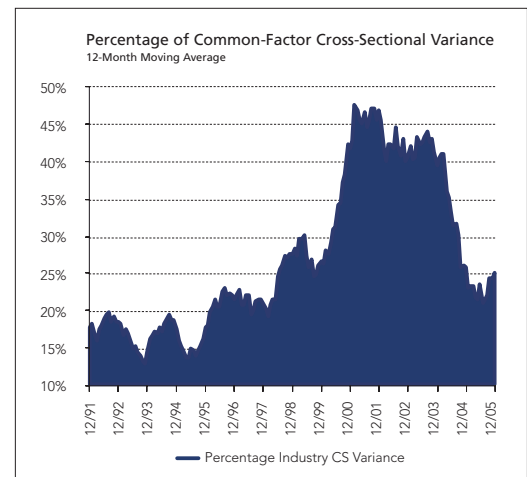
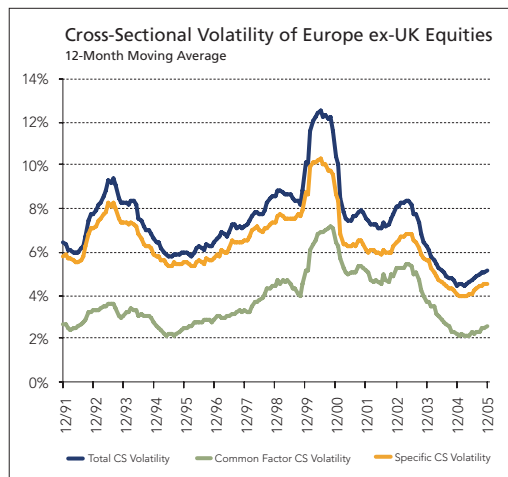
The implication of this decline for portfolio management is significant as managers with industry-neutral, style-based active strategies will have little scope to distinguish themselves from their peers. Conversely, there is more scope for managers who focus on industry selection.

### Continental European Equity Market, 1992 to 2005

After the bursting of the internet bubble, in a pattern similar to that in the US market, CSV in Europe dropped rapidly in 2000 and 2001, and even further in 2004 (see Figure 4). As in the US, both stock-specific and common-factor CSV decreased at a similar pace.

While the picture looks similar to that in the US, a closer look reveals interesting differences, in particular during 2005 when CSV in Europe began to increase again.

Figures 4 and 5



Moreover, the sources of CSV—common-factor and specific returns—behaved differently than in the US. As seen in Figure 5, we witnessed a dramatic increase in the importance of common-factor CSV towards the end of the bubble, with common factors accounting for more than 40% of individual stock-return variation between 2000 and 2003. Then in 2004, common-factor CSV dropped as rapidly as it had increased in 2000. It is now again at late 1990s levels. This suggests that in just the last two to three years, stock pickers managing very concentrated portfolios have experienced greater return

dispersion compared to their more diversified peers.

As in the US, among the common factors, industries increased in importance relative to style (and country)<sup>3</sup> factors over the last 4 years, recovering from very low levels (less than 50%) in early 2002 (see Figure 6)<sup>4</sup>. Interestingly, only a few risk indices account for most of the relative decrease of industry CSV during 2000 and 2001: Market Sensitivity<sup>5</sup>, Momentum, Size and Volatility. In particular Market Sensitivity showed consistently large negative or positive returns during 2001.

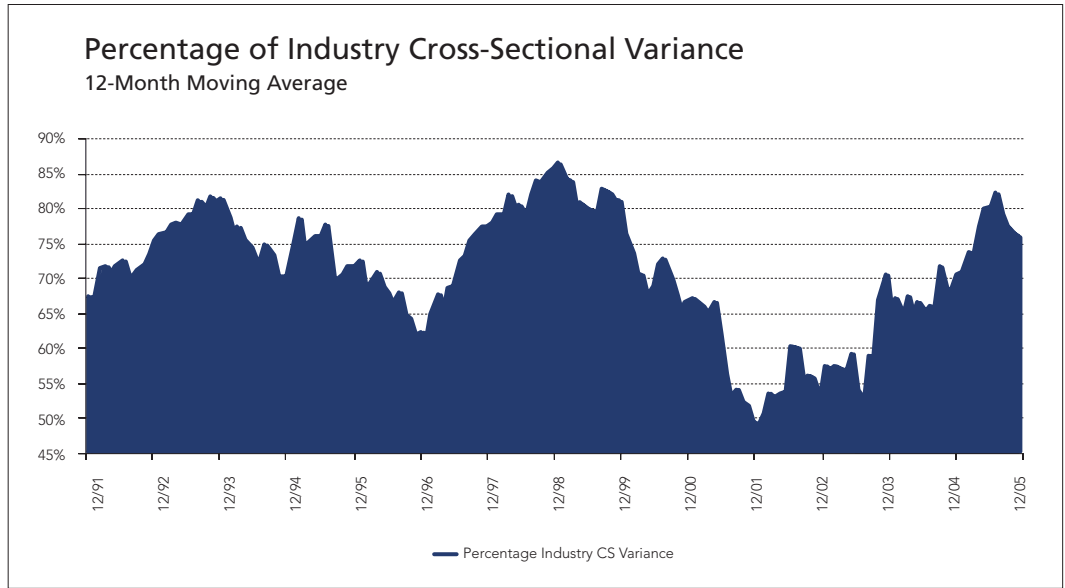
<sup>3</sup> There are also country factors in the Europe Equity Model (EUE2). The market return is captured by the industry factors.

<sup>4</sup> The European Equity model, EUE2, is a regional model that utilizes not only on regional industry and style factors but captures the country specific commonalities through a set of country factors. The model is described in 'Forecasting European Equity Risk over Different Horizons', Nicholas Herbert, MSCI Barra White paper, 2005.

<sup>5</sup> Market Sensitivity records the historical exposure of a stock's return to movements in the overall market, based on a 5-year historical beta regression. When markets experience large swings in either direction the estimated Market Sensitivity return will also be quite large.

Figure 6

Industry factors now explain about 75% to 80% of common-factor CSV.

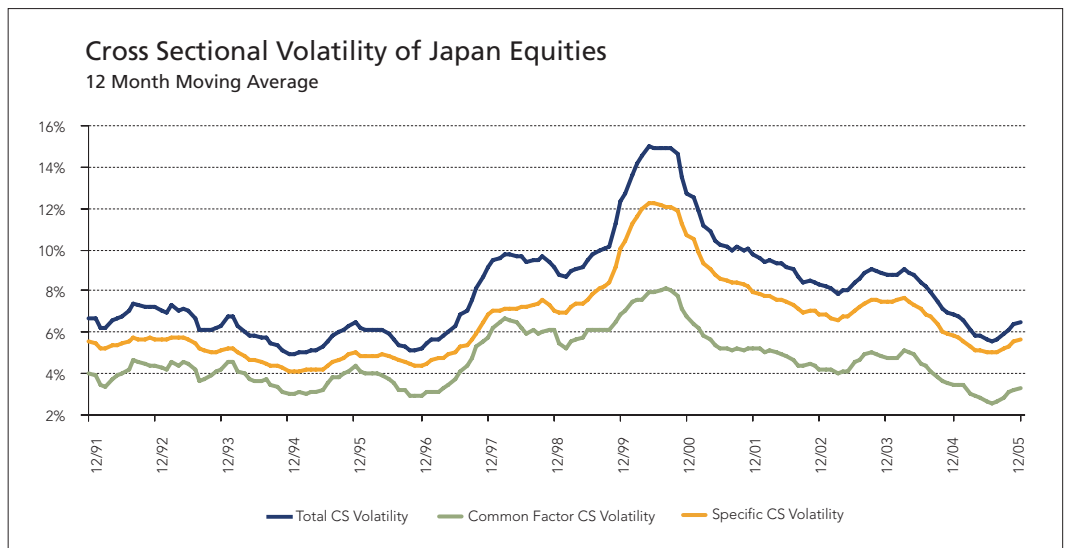


*The Japanese Equity Market, 1992 to 2005*

Figure 7, which charts CSV in the Japanese market, looks similar to the US and Europe (Figures 1 and 4). CSV has decreased steadily since 2001 and is now at levels last seen in 1995.

As in Europe, CSV has picked up moderately in 2005, making it more likely that tracking errors will increase and the returns of active managers will be more dispersed.

Figure 7

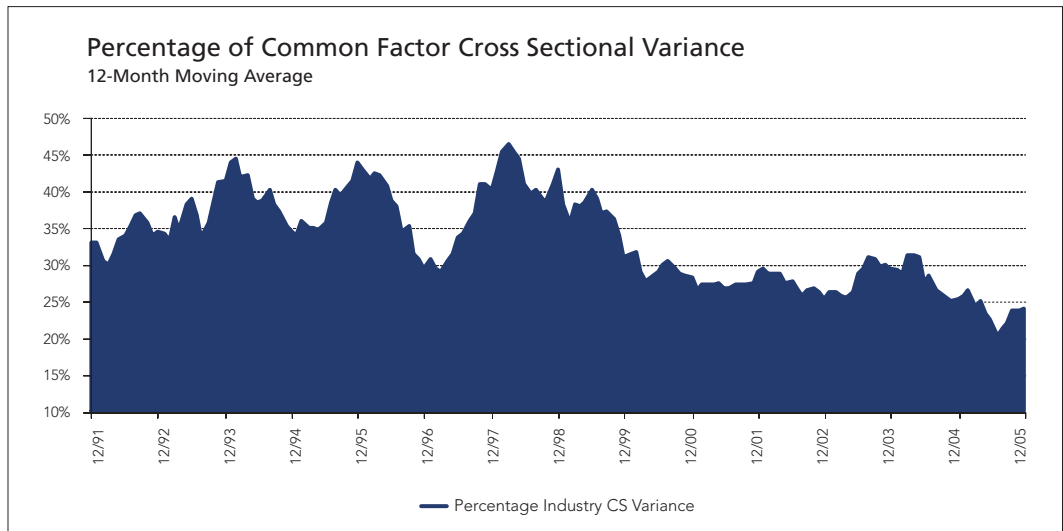


As seen in Figure 8, the relative importance of common factor variation versus specific return variation in the Japanese equity market paints a very different picture from that of the US or Europe. Between 1998 and 2000, common factor-driven stock return variation rapidly decreased.

It further decreased in 2004 to levels not seen in 15 years before picking up again in 2005.

The implications of this decline in common factor CSV for Japanese equity managers are significant. Active managers investing in concen-

Figure 8



trated portfolios, i.e., running significant active bets focusing on stock specific events, are likely to have distinguished themselves significantly from their peers; more so than diversified active strategies focusing on industry or style bets.

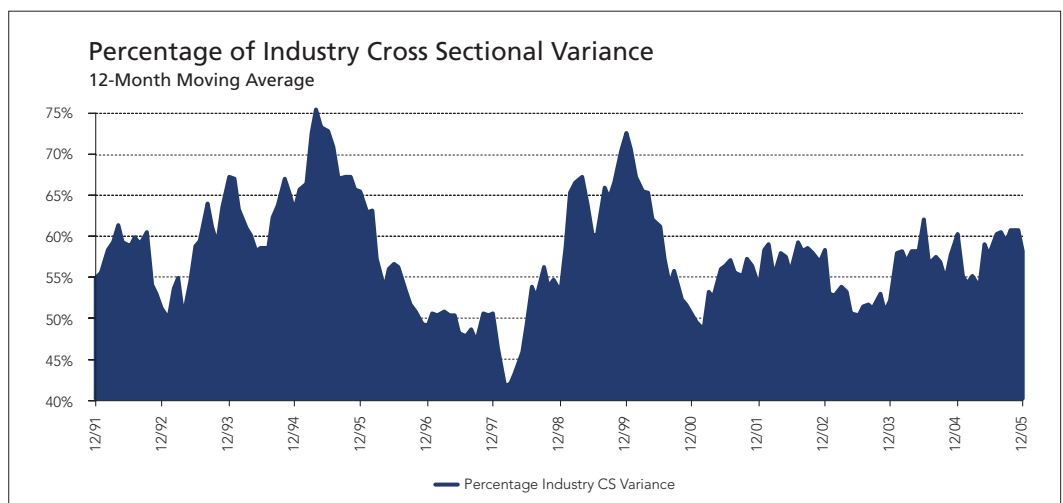
The picture for Japan again appears very different from the US and Europe when we look at the composition of common factor driven return dispersion. As plotted in Figure 9, industry driven CSV plummeted to just 40% in 1997/1998. This decline occurred much earlier and was more severe than in the US and Europe.

Over the last 5 years, the percentage of common factor return dispersion explained by

industries has fluctuated only between 50 and 60%. Risk indices in Japan seem to play a much more significant role for active managers than in the other markets.

In particular, Momentum, Volatility, and to a lesser degree, Size, have shown both strong positive and negative returns over the last 15 years. Active managers ought to be conscious about their risk index exposures in Japan and monitor them closely. Managers with active risk index bets in Japan have significantly more potential to distinguish themselves from their peers than in the US and Europe.

Figure 9



### Implications for Portfolio Management

As we have seen, cross-sectional volatility in major markets has experienced significant swings over the last decade. Alongside these swings in overall CSV, the composition of common factor CSV has also varied markedly. Investment managers need to be cognizant of these shifts when they are structuring their portfolios and managing tracking error.

To quantify the impact of these variations in CSV on portfolio management we simulate the following two strategies in each of our three markets:

1. A passive strategy replicating an index with a limited number of securities while minimizing tracking error on a monthly basis. This strategy is characterized by fairly stable active factor exposures. We examine the evolution of tracking error over time for the strategy as well as the relative changes in the sources of risk, i.e., the dynamics of common factor and asset specific contribution to tracking error.
2. An active strategy with a value bias and a tracking error target of 4%. We optimize with a set of expected returns based on the Barra VALUE Risk index exposures of the investment universe, again on a monthly basis. We expect to see a varying number of securities and significant shifts in active exposures driven by the dynamic levels of CSV.

The first (passive) strategy isolates the impact on tracking error of CSV variations. If tracking error changes are largely associated with fluctuations in CSV, then we can argue that a certain amount of tracking error is conditional on the level of CSV. The second (active) strategy isolates the impact of CSV on portfolio composition when a tracking error target is a primary management objective. We expect that if CSV

changes, the set of active bets must adjust to stay within the defined portfolio tracking error range. Moreover, the optimal number of securities will vary along with CSV levels. Both effects can cause unintended changes in the portfolio's return to risk profile.

### Passive Replication with a Limited Number of Securities

For the first strategy, we optimize against benchmark indices in the US (S&P500), Japan (MSCI Japan), and Europe (MSCI Europe). The objective is to minimize tracking error relative to the index with a maximum number of 75 stocks. The securities in the respective market index form the universe for each optimization. We then create monthly portfolios using our optimizer with relatively stable active factor exposures. Asset specific risk is diversified and is mostly a function of the number of securities, 75, in our case study.

We first examine the evolution of the monthly tracking error forecast over the period from January 1997 to December 2005 (Figure 10). Significant fluctuations in tracking error may cause concern for passive or enhanced index strategies whose main objective is to stay close to an index. Unfortunately, all three country portfolios experience major changes in tracking error over this time period.

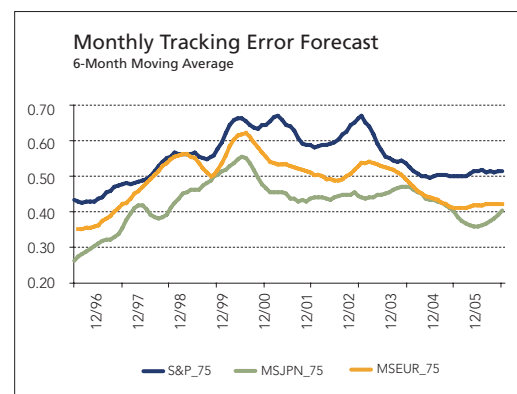


Figure 10

Starting in late 2000 the tracking error exhibited by the Japanese and the European portfolios began to fall steadily, a trend which continued over the next 4 years. Meanwhile, the active risk of the US portfolio remained high up until early 2003 before declining rapidly over the subsequent 12 months. In 2005, the tracking error of the European and the US portfolios stabilized while that of Japan's

increased slightly. All three portfolios' tracking error ended the year at relatively low levels, on par with 1998 levels.

How much of these changes in tracking error have been driven by the dynamics of CSV? To understand this relationship, we calculate correlations between the monthly tracking errors and CSV for each of our markets.

Table 1

| Correlation    | S&P_75 | MSEUR_75 | MSJPN_75 |
|----------------|--------|----------|----------|
| 1/1997-12/2005 | 0.71   | 0.53     | 0.57     |
| 7/2001-12/2005 | 0.73   | 0.74     | 0.66     |
| 1/1997-6/2001  | 0.84   | 0.41     | 0.59     |

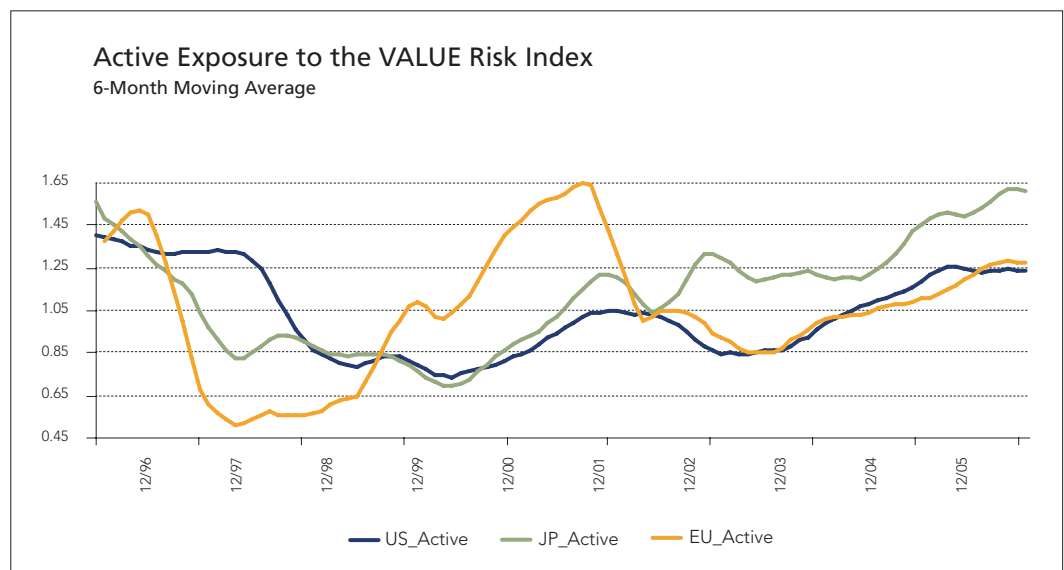
Table 1 confirms there exists a strong relationship between CSV and tracking error across all three portfolios. Overall, the relationship is strongest for the US portfolio with correlations consistently greater than 60% since January 1997. Correlations for the European portfolio are a great deal higher during the most recent 4.5 years, surpassing that of the US portfolio. The Japanese portfolio correlations are the lowest and, like the US portfolio, appear to be stable through time.

In sum, our passive strategy replicating an index with a limited number of securities experiences significant swings in tracking error over time. These changes appear to be closely related to the level of CSV.

*Active Value Strategy with a 4% Tracking Error Target*

Our second simulated strategy is an active value-biased strategy with a preset tracking error target of 4% against the same set of

Figure 11



market indices we used in the passive strategy simulation. Here again, we treat the stocks within the respective indices as the asset universe. We allow the size of active bets and the number of securities to vary. We expect to see more concentrated holdings as well as an increase in active bets in times when CSV is low and opportunities to generate alpha are more limited.

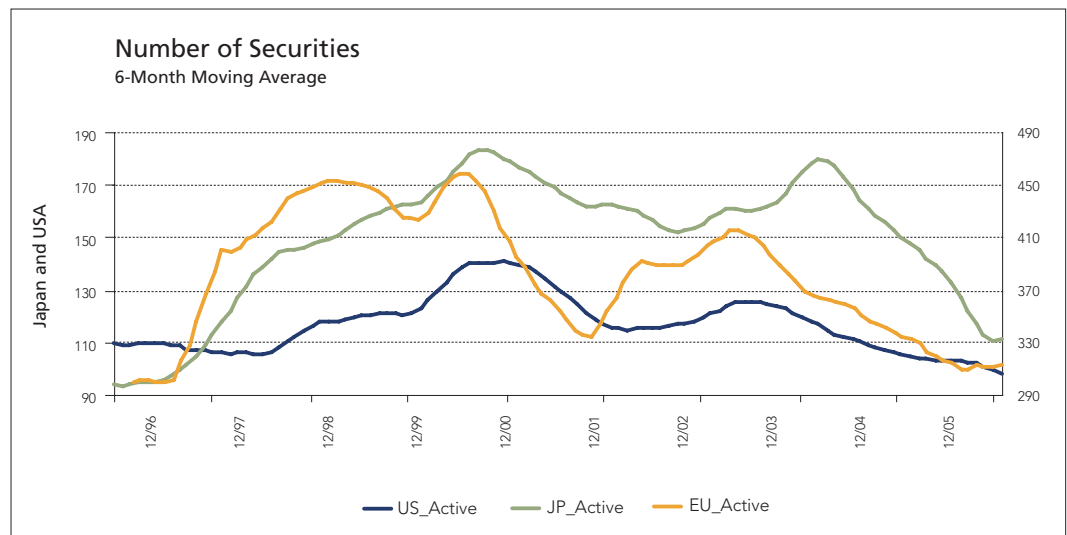
Figure 11 shows that active exposure to the VALUE Risk Index increases in all three portfolios from 2002-on accompanied by a decrease in both market volatility and CSV. Compared

to the other two portfolios, the European portfolio exhibits much larger swings in its active exposure to the VALUE Risk index.

The number of securities required to achieve the 4% tracking error target gives us an idea about how flexible strategies with a given tracking error target need to be.

For all three strategies, the number of securities in the optimized portfolios peak at the height of CSV in 2000 and decrease in tandem with levels of CSV and market volatility from 2002/2003-on.

Figure 12



### Summary of Implications for Portfolio Management

Our two naïve strategies highlight the potential impact of dynamically changing levels of CSV on portfolio strategies. Investment mandates with a relatively fixed number of holdings are likely to experience wide fluctuations in tracking error. Maybe even more importantly, such strategies have to accept stark variation in expected (and realized) alpha.

Meanwhile, investment mandates with narrow tracking error targets need to adjust both their

active exposures and the number of securities in the portfolio significantly to stay within the defined range. Those strategies that increase active bets in times of declining volatility imply either an increased confidence in (lower) return forecasts or assume that returns will stay at levels seen in higher volatility periods.

Should the confidence in return forecasts increase as volatility declines? Unfortunately, there is no universal response to this question as the answer depends on the investment strategy and the source of decline in volatility.

If the cause of the decline is mostly driven by a plunge in asset specific return variations, managers focusing on sector or style driven strategies will be delighted. They should have more confidence in their expected sector or style alphas as the noise (asset specific returns) has dwindled. Increasing active bets under this scenario should lead to an increased risk adjusted return (on average over all such managers). The likely losers are the stock pickers, as their abilities to generate active returns have lessened.

Finally, the rapid decrease in volatility and expected returns over the last few years has led to an increased focus on transaction costs and management fees as their relative importance increased with shrinking alpha.

Not surprisingly, optimizing trade execution to control transaction costs and market impact has gained momentum. Trading algorithms for best execution are mushrooming all over financial centers and some investment management firms are developing 'in-house exchanges' to avoid market impact altogether.

### Conclusion

cross-sectional volatilities in major markets have experienced significant swings over the last decade. Alongside the swings in overall CSV, the composition of common factor CSV has shifted. Active managers need to be cognizant of these shifts in terms of structuring their portfolios and managing tracking error. ■

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