Overview

Most asset owners have traditionally viewed fixed income securities as a relatively safe asset class haven from volatility in equity and other markets. While it is certainly true that government bonds are generally a low risk asset class for domestic investors in developed markets, long-term government bonds can be every bit as risky as a diversified equity portfolio. More generally, many fixed income securities, such as mortgage backed securities (MBSs), collateralized debt obligations (CDOs), or high-yield bonds can be relatively risky investments.

Driven by a variety of pressures, including requirements from asset owners and regulators, there is a continuing demand in the financial community for improved tools for quantitative risk forecasting of fixed income portfolios. Risk analysis is the art and science of forecasting portfolio return variability. It involves several components. One is the choice of the risk measure. Typical in the asset management community is use of the width of the expected return distribution, commonly the standard deviation. An alternative, widely used in the banking world, is a loss value measure such as value-at-risk (VaR), but this is less relevant for asset managers who are more concerned with return measures and performance relative to a benchmark. ¹

A second component is the method of forecasting a portfolio return distribution. Standard approaches include using the historical return distribution for portfolio assets to estimate their future return distribution and using a factor model to characterize asset returns, together with a model for predicting future factor return distributions. Although sometimes used in the equity world (though it suffers when attempting to scale to large portfolios), the first approach is not useful in the fixed income world because there are very obvious market factors affecting all assets, because of the very large number of individual securities (in the millions), and because of the finite lives and time dependent characteristics of the assets.

This paper will focus on fixed income risk modeling using factor models to forecast portfolio return standard deviation. Conceptually, this is a relatively straightforward problem, although as with many other aspects of life in the bond world, practical implementations are full of challenges. Our focus is primarily on forecasting at the intermediate horizon of one month. However, most, if not all, of the ideas presented here are applicable both at longer and shorter horizons. Forecasting at daily or even intraday horizons presents significant challenges with respect to problems such as timing and synchronization, and is beyond the scope of this paper.

¹ VaR has also come in for significant criticism on the grounds of, among other things, failure to have good additive properties, and failure to measure the magnitude of expected loss and above the VaR threshold. See Philippe Ritzner, Freddy Delbaen, Jean Marc Eber, and David Health, "Coherent Measures of Risk," Mathematical Finance 9(3) (1999), pp. 203-228.
The paper is organized as follows. The first section describes in detail a general framework for analyzing the risk of portfolios of fixed income securities. In the following sections, we discuss each risk component individually, and then present a method to aggregate components and create a global risk model. The last section shows the risk of several typical standard benchmarks.

**Modeling Framework**

Understanding and forecasting risk accurately consists in identifying the factors that drive the price of securities in the marketplace and adequately capturing these factors in a model. We observe historical asset returns, and our challenge is to explain them in terms of a minimal set of explanatory market factors, whose return distribution (along with that of the residual asset (returns) then serves as the basis for asset or portfolio risk forecasts.

The task of return attribution is to identify a set of common factors, whose changes \( f_t \) “explain” the excess returns (returns over the risk-free rate) \( r_t \) of the assets we are concerned with. In general, there is considerable arbitrariness in the identification of the factors, but some choices are more natural or straightforward than others. Return attribution then amounts to solving by regression the relationship

\[
r_t = X_t \cdot f_t + \varepsilon_t
\]

This equation states that the excess return to asset \( k \) over a period starting at time \( t \) is equal to the dot product of the common factor returns \( f_t \) with the asset’s exposure to each, \( X_t \), plus a residual asset-specific return \( \varepsilon_t \). Note that the common factor returns \( f_t \) do not depend on the asset. Given the assets’ exposures (which may be time dependent), we can solve by regression for the \( f_t \) to minimize the size of the unexplained residuals \( \varepsilon_t \).

Many of the factors driving bond returns can be understood by examining the basic valuation formulas or algorithms. The simplest arbitrage free model of security valuation, applicable to default-free bonds with fixed cashflows serves as a useful starting point for understanding more detailed models. The bond value is the sum of cashflows present values with discount rates given by the term structure of interest rates:

\[
P = \sum_{i=1}^{N} CF_i e^{-r_i t_i}
\]

The present value \( P \) of a bond is the sum of cashflows \( CF_i \) at times \( t_i \) discounted by the interest rates \( r_i \).

For a fixed-coupon Treasury bond, the cashflows are the coupon, and the interest rates are the prevailing risk-free rates. The valuation formula becomes more complex as soon as we leave the realm of plain vanilla government securities. In the general case, the cash flows are not known in advance and may be state or even history dependent, and the discount factors include a spread and must be computed pathwise. The spread is a shorthand means to capture the excess return required by investors to compensate for various risks, most importantly default and liquidity.

In general, for risk modeling purposes, we can take the valuation model as a black box with various inputs, such as the term structure, spread, volatility forecast and prepayment model, and derive risk forecasts without further reference to the model details. (Of course, this is predicated...
on someone having built a good valuation black box that can be relied on to take all the necessary inputs and provide an accurate present value output.) The inputs for equation (X.2) and more complicated valuation models are useful for identifying the sources of market risk. One immediately sees from equation (X.2), for example, that a government bond is exposed to risk factors defined by changes in interest rates at different maturities. We will discuss the various sources of risk in more detail in the next sections.

A detailed understanding of correlations between asset returns is also required to accurately estimate the risk of a portfolio. Estimating correlations directly is in practice impossible as unknowns severely out-number observations even in relatively small portfolios. Fortunately, the factor attribution of equation (X.1), allows us to model the asset return correlations in terms of a relatively small number of factor covariances.

Since, by construction, factor and specific returns are un-correlated, and since specific returns are also un-correlated with each other (leaving aside the correlation of bonds from a common issuer):

\[ \sigma^2 = h^T \Sigma \cdot h \]  

(X.3)

with

\[ \Sigma = T X \cdot \phi \cdot X + \Delta \]  

(X.4)

where

- \( h \) = the vector of portfolio holdings
- \( \Sigma \) = the covariance matrix of asset returns
- \( \phi \) = the covariance matrix of factor returns
- \( \Delta \) = the diagonal matrix of specific variances

Equation (X.4) will yield active risk forecasts when \( h \) is a vector of active holdings, that is when the portfolio weights are relative to those of a benchmark.

The data that can go into computing factor returns will of course depend on what the factors are. It may include bond and index level data as well as currency exchange rates. Given a set of factor return series, we seek a forecast of the factor covariance matrix. The simplest approach is to use the sample covariance matrix of the full return history. If the underlying return-generating process is fixed, that is time independent, this is an optimal estimator. In practice, however, this condition is unlikely to be met: external circumstances change, markets change, and it seems reasonable to expect the dynamics of the term structure to vary in time. Forecasts based on equal weighting of historical data gradually become less and less sensitive to the arrival of new information. Although the forecasts are extremely stable (which can be an attractive feature), the price of this stability is that the forecasts become non-responsive to changes in the dynamics.

A simple method for addressing this variation is to weight recent returns more heavily than older ones in the analysis, with weight proportional to an exponential of the age of the data. The weight of returns from time \( t \) in the past relative to the most recent returns is \( e^{-\tau/t} \), where \( \tau \) is the time
scale\textsuperscript{2}. The optimal time constant $\tau$ can be obtained empirically using, for instance, a maximum-likelihood estimator. However, particularly volatile series may benefit from a different treatment (see the Currency Returns section).

This is our multi-factor framework for forecasting risk. Note that factors are descriptive and not explanatory. In other words, they permit one to forecast risk without necessarily being identified with the underlying economic forces that drive interest rates or bond spreads.

We now proceed with an identification of the factors and the calculation of their returns.

**Interest Rate Risk**

Interest rate or term structure risk arises from movements in the reference, or “benchmark” interest rate curve. If we exclude currency risk, it is the dominant source of risk, at least for most investment-grade bonds. Building a term structure risk model entails first choosing the benchmark curve. Domestic government bond yields are the choice in most markets, but there are important exceptions that can lead to some complications.

The euro zone presents a particularly complex picture. On the one hand, the LIBOR/swap curve has emerged as the preferred benchmark for corporate debt due to the absence of a natural government yield curve and the development of a liquid swap market. On the other hand, domestic government debt continues to trade relative to its local government benchmark, and although yields have converged, some differences clearly remain that invite choosing a different government benchmark curve in each legacy market. Overall, the benchmark curve is debt-type and country dependent.

In some smaller markets, the absence of a liquid market for government debt makes the LIBOR/swap curve the only available benchmark. In a few extreme cases such as in markets affected by extremely high inflation, there is little reliable interest rate data and the best we can do is come up with some reasonable short interest rate.

As long as common factors accurately describe (1) interest rate risk and (2) risk with respect to the benchmark, risk forecasts are in fact not benchmark-dependent. Yet, selecting a sub-optimal benchmark may limit our ability to correctly identify and hedge a critical factor. For instance, even in markets where securities are quoted off the swap curve, changes in government yields are the dominant underlying source of risk. This is not as clear when interest rate risk is expressed with respect to the swap curve. One approach is to use the government term structure as local benchmark whenever possible and include a swap “intermediate” factor that can be added to the government-based interest rate factors to allow interest rate to be expressed with respect with the swap curve. This swap factor will be described in more detail in the next section. In markets where the benchmark is already the LIBOR/swap curve, there is obviously no need for a swap factor.

Within a given market, as defined by the currency, it may not be appropriate to value all bonds in relation to a single benchmark. This is the case for US municipal bonds, which, thanks to their tax-exempt status, trade at prices affected by various tax rates as well as by the issuer’s

\textsuperscript{2} The half-life is then $\sigma \ln 2$. 
creditworthiness. It is also the case for inflation protected bonds (IPBs), which offer investors a “real” inflation-adjusted yield. Such securities are weakly correlated with other asset classes and require IPB-specific real yield risk factors.

What should the interest rate factors be? Key rate factors, which are rate changes at standard maturities e.g., 1, 3 and 6 months and 1, 2, 3, 5, 7, 10, and 30 years seem a natural and somewhat appealing choice. However, because changes in rates for different maturities are highly correlated, using so many factors is unnecessary. Correlations of interest rate changes approach 1 for nearby maturities and are positive between all maturities in all markets we have studied. This is a consequence of there being a dominant, approximately maturity independent factor driving the changes in the key rates. Using principal component analysis (that is, extracting the eigenvectors of the covariance matrix of the spot rate changes \( \Delta s(t, T) \)), we find that this leading principal component together with the next two account for about 98% of the key rate covariance matrix (the exact fraction depending on the market). That is, reconstructing the key rate covariance matrix from just these three factors leaves an average fractional error in the matrix elements of around 2%.

**Exhibit 1 — US Dollar Interest Rate Risk Factor Shapes**

Based on their shapes, shown in Exhibit 1 for the US government bond yield curve, the three principal components are referred to as “shift”, “twist” and “butterfly” (STB). In this case, equation (X.1) takes the form

\[
 r^I_k = \sum_{i \in S, T, B} D^I_k \cdot r^I_{STB, i} + \varepsilon^I_k \tag{X.5}
\]
where the $S$, $T$, and $B$ “durations” of bond $k$, while $r^I_{STB,i}$ are the $S$, $T$, and $B$ factor returns.

Typical Shift, Twist and Butterfly volatilities are shown in Exhibit 2. Shift-like changes are the dominant source of risk in all cases with annualized volatilities ranging from roughly 40 to over 400 bp/yr. Aside from differences of scale, the character of term structure risk is relatively homogeneous across most major markets. A rule of thumb is that twist volatilities are usually about half of shift volatilities, while butterfly volatilities are in turn half of the twist volatilities. Not surprisingly, the largest volatilities are observed for emerging markets such as China, and IPB real yield curves are less volatile than their nominal government counterparts.

**Exhibit 2 — Interest Rate Factor Volatilities on December 31, 2004**

The interest rate risk of any given bond will depend first on the bond’s exposures to the factors, and to a much lesser degree on correlations between factors\(^3\). Exhibit 3 gives examples of risk decompositions for three sovereign bonds. The annualized risk of a straight bond issued by the US Treasury varies from about 1% to over 10%, depending on its duration. Korean domestic government bonds have comparable risk characteristics.

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\(^3\) This is because principal components are, by construction, only weakly correlated. (They are not uncorrelated because we estimate their returns by regression on bond returns than from the key rate returns.)
Exhibit 3 — Examples of Interest Rate Risk Breakdown on December 31, 2004

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Risk (bp/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shift</td>
</tr>
<tr>
<td>US Treasury</td>
<td></td>
</tr>
<tr>
<td>5.25% 11/15/2028</td>
<td>9.9</td>
</tr>
<tr>
<td>US Treasury</td>
<td>1.0</td>
</tr>
<tr>
<td>1.625% 02/2/2006</td>
<td></td>
</tr>
<tr>
<td>US Treasury</td>
<td>6.7</td>
</tr>
<tr>
<td>4.75% 09/17/2013</td>
<td></td>
</tr>
</tbody>
</table>

Spread Risk – The Conventional Approach

International bond portfolios were not long ago still mostly composed of government bonds. The recent explosion of the global corporate credit market now provides asset managers with new opportunities for higher returns and diversification. Unlike government debt, however, corporate debt is exposed to credit and liquidity risk, which are manifest as changes in valuation relative to the benchmark yield curve. For modeling purposes, such changes can again be decomposed into a systematic component that describes, for instance, a market-wide jump in the spread of A-rated utility debt and can be captured by common spread factors, and a bond-specific component. This section discusses market-wide spread risk, while the next section will address specific spread risk and default risk.

Data considerations are crucial in choosing factors. We can virtually always construct term structure risk factors, whereas spread factors are more data-dependent. In other words, the choice of factors will be limited in markets with little corporate debt. Spread factors should increase the investor’s insight and be easy to interpret. Meaningful factors will in practice be connected to the portfolio assets and construction process and allow a detailed analysis of market risk without threatening parsimony.

Swap Spread Factors

In markets with a government bond benchmark yield curve, the spread of LIBOR and swap rates over government rates provides a useful measure of the combination of a liquidity premium on government bonds and the market price of the credit risk on high grade debt (generally taken as
equivalent to a AA agency rating). In markets with a corporate bond market that is not deep or transparent enough for estimation of a detailed credit risk model, we can use this LIBOR/swap spread as a proxy factor for modeling risk of high-grade bonds relative to the government curve. Given the high correlation of credit spreads of high-grade issuers, this single factor model is a reasonable approximation. We can also account for the greater risk of lower quality issuers by using the ratio of bond spread to LIBOR/swap spread as a measure of exposure to the swap risk factor. Linear dependence turns out to overestimate the spread risk of lower quality bonds, but a sub-linear power law generally does a fair job across the markets where we do have more detailed corporate bond data for comparison.

Swap spread volatilities for several currencies are shown in Exhibit 4, with values that vary from about 15 bp/yr to 40 bp/yr. Also shown are the resulting spread risks in the euro and sterling markets for several rating categories. We will see further below that the swap model predicts reasonably accurately both the absolute magnitude of the spread risk in each market and their relative values.

In many emerging markets, the swap curve is the benchmark and we cannot build a swap spread factor. We need a reasonable alternative basis for a simple spread risk model.

One natural approach would be to replace the swap spread by an average credit spread derived from a representative set of domestic corporate bonds. In practice, liquidity issues make this apparently simple scheme hard to implement. There are often a very limited number of outstanding corporate bonds available in each market. Because many of them are infrequently traded, a model builder is not in a position to obtain accurate prices in these illiquid markets.

An alternative approach is to construct the factor from a universe of arguably more liquid external debt. Consider for instance Asian emerging markets; there are at least two indices that track the performance of Asian US dollar-denominated debt with respect to the US sovereign benchmark: HSBC’s Asian US Dollar Bond Index (ADBI) and JP Morgan’s Asian Composite Index (JACI). Although these indexes track external debt, whereas we are interested in domestic debt, some simple considerations suggest that they may still be useful for the purpose of deriving an average measure of domestic spread risk.

Spreads between corporate and benchmark yields compensate investors for credit risk, liquidity, and, in some cases, disparate tax treatments. In practice, emerging market spreads are mostly determined by credit risk considerations. Creditworthiness is attached to the issuer and is to a large extent independent of the market on which a bond is issued. As a result, differential credit spreads between two issuers will also be market-independent and we can write for instance:

\[ S_{\text{Yuan, CM}} - S_{\text{Yuan, Gov}} = S_{\text{US$, CM}} - S_{\text{US$, Gov}} \]  

(X.6)
Exhibit 4 — Examples of Annualized Swap Spread Volatilities on December 31, 2004

Comparison Between Typical Euro and Sterling Spread Volatilities Computed Using the Swap Factor for Different Rating Categories

where

\[ s_{\text{Yuan,CM}} \] (respectively \( s_{\text{Yuan,Gov}} \)) is the spread of a bond issued by China Mobile (respectively the Chinese government) on the Chinese domestic market

\[ s_{\text{US$\text{-CM}}} \] (respectively \( s_{\text{US$\text{-Gov}}} \)) is the spread of a US dollar denominated bond issued by China Mobile (respectively the Chinese government).
Although not all issuers are simultaneously active on both the Euro dollar market and their domestic market, we can extend equation 6 to all issuers with comparable characteristics so that

$$S_{\text{Yuan, Corp}} - S_{\text{Yuan, Gov}} \approx S_{\text{US$, Corp}} - S_{\text{US$, Gov}}$$ (X.7)

where $S_{\text{Yuan, Corp}}$ (respectively $S_{\text{Yuan, Gov}}$) is the average spread of investment grade corporate bonds (respectively Chinese government) bonds on the Chinese domestic market and $S_{\text{US$, Corp}}$ (respectively $S_{\text{US$, Gov}}$) is the average spread of US dollar denominated bonds issued by Chinese investment grade companies (respectively the Chinese government).

$S_{\text{US$, Gov}}$ is typically reported in an index such as JACI or ADBI. A quasi-sovereign spread can be used when the sovereign spread is not available. It is also possible to derive $S_{\text{US$, Corp}}$ from one or more subcomponents of the same indices. In other words, we can construct a proxy for the local credit spread factor using a global emerging market credit index and, as described earlier, scale it to account for varying credit qualities.

**Detailed Credit Spread Factors**

Accurately modeling spread risk in major markets such as the US dollar or Japanese yen market requires detailed currency-dependent, “credit blocks”.

Various considerations drive the choice of spread factors. Factors built on little data can end up capturing a large amount of idiosyncratic risk and be representative of a few issuers rather than the market. A corollary is that it is often wiser to avoid building separate factors for thin industries. Spread factors should be meaningful for the investor, and be related to the process of constructing a portfolio.

A simple and natural approach is to capture fluctuations in the average spread of bonds with the same sector and rating. There is unfortunately not enough data to construct sector-by-rating factors for all low-grade ratings and the simplest alternative is then to construct rating-based factors. A typical sector and rating breakdown for the euro and US dollar markets is given in Exhibit 5.

A non-domestic sovereign bond is exposed to the factor corresponding to its sector and rating. Due to the limited number of high-yield bonds outstanding, some non-investment grade factors are only broken down by ratings.
Exhibit 5 — Sector and Rating Breakdown in the Euro and US Dollar Spread

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Ratings</th>
<th>Sectors</th>
<th>Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic agency</td>
<td></td>
<td>Domestic agency</td>
<td></td>
</tr>
<tr>
<td>Agency</td>
<td>AAA</td>
<td>Energy</td>
<td>AAA</td>
</tr>
<tr>
<td>Financial</td>
<td>AA</td>
<td>Financial</td>
<td>AA</td>
</tr>
<tr>
<td>Foreign sovereign</td>
<td>A</td>
<td>Foreign agency and local</td>
<td>A</td>
</tr>
<tr>
<td>Energy</td>
<td>BBB</td>
<td>Industrial</td>
<td>BBB</td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
<td>Foreign sovereign, Supranational</td>
<td>BB</td>
</tr>
<tr>
<td>Pfandbrief</td>
<td></td>
<td>Telecom</td>
<td>B</td>
</tr>
<tr>
<td>Supranational</td>
<td></td>
<td>Transportation</td>
<td></td>
</tr>
<tr>
<td>Telecom</td>
<td></td>
<td>Utility</td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td></td>
<td></td>
<td>CCC</td>
</tr>
</tbody>
</table>

Note that using market-adjusted ratings as opposed to conventional agency ratings can increase the explanatory power of sector-by-rating spread factors. The idea is to adjust the rating of bonds with a spread that is too different from the average spread observed within their rating category. For instance, a AA-rated euro-denominated bond with a spread equal to 200 bp would be reclassified as having an implied BBB rating.\(^4\) Credit spreads are computed with respect to the local swap curve to accommodate for the swap spread factor.

Arbitrage considerations indicate that the spread risk of issues from the same obligor should be independent of the market. Why then do we need different credit factors for the different markets? After all, a model with only one set would be more parsimonious. Empirical evidence simply shows that spread risk is indeed currency-dependent.\(^5\)

Volatilities for selected factors are displayed in Exhibit 6. Spread risk in the Euro and US markets is on average quite different, particularly for high-grade securities. Looking now in more detail, sterling factors tend to be more volatile than Euro factors for AAA, AA and A ratings, and less volatile for lower ratings. This is a trend already seen that confirms that the swap factor would be a simpler but meaningful alternative.

\(^4\) For further details on this point, see Ludovic Breger, Lisa Goldberg and Oren Cheyette, “Market Implied Ratings.” \(Risk\) (July 2003), pp S21-S22

Exhibit 6 — Euro and US Dollar Spread Factor Volatilities as of December 31, 2004

Significant differences exist for individual factors that illustrate the need for currency-dependent factors (see for instance the Telecom A and BBB factors). Also note how the high volatilities of the Energy, Utility and especially Telecom factors reflect the recent turmoil in these industries.

Each corporate bond will only be exposed to one of these factors, with an exposure equal to the spread duration. For a fixed-rate bond, this will generally be numerically close to the shift factor exposure. Empirically, the spread risk of almost all AAA, A, and A-rated bonds will be less than their interest rate risk, and it is only for BBB-rated bonds and in some very specific market sectors such as Energy and Telecoms that spread risk becomes comparable to or exceeds interest rate risk. Spread risk is the dominant source of systematic risk for high-yield instruments.

**Emerging Markets Spread Factors**

Emerging debt can be issued either in the local currency (i.e., Croatia issuing in kuna) or in any other external currencies (i.e., Mexico issuing in euro, sterling, or US dollars). These two types of debt do not carry the same risk, and need to be modeled independently. "Internal" risk was discussed in the interest rate and swap spread risk sections and we will now address external risk.

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6 External debt is more risky than internal debt. In principle, a national government can raise taxes or print money to service its internal debt. A shortage of external currency can be more of a problem. This is reflected in the agency credit ratings for emerging market issuers.
A rather natural approach is to expose emerging market bonds to a spread factor. The sovereign spread factor turns out to be a poor candidate as the risk of emerging market debt strongly depends on the country of issue. Exhibit 7 shows average Argentinean monthly spread changes from June 30, 1999 to June 30, 2002 for US dollar-denominated debt. The collapse of the peso, the illiquidity of the financial system and other economic fallout are all reflected in Argentinean spreads. Chilean spreads remained virtually unaffected despite a strong economic link between the two countries. As a result, any accurate model will need at least one factor per country of issue.

The amount of data available for building emerging market spread factors is unfortunately rather limited. First, there are often at best only a few bonds issued by sovereign issuers in emerging markets. The second problem is that these are mostly US dollar-denominated. Even when some bonds denominated in, say, Euro are available, there is generally little returns history. In some cases, a risk model will even be asked to forecast the risk of obligors that just started issuing in a specific currency. Since the risk of an emerging market bond is directly related to the creditworthiness of the sovereign issuer, which is independent of the currency of denomination, we can actually borrow from the history of US dollar-denominated emerging market returns to forecast spread volatilities in any major currencies. Spread return data can be obtained from an index such as JP Morgan Emerging Markets Bond Index Global (EMBIG).

Exhibit 7 — Examples of Spread Returns for Two Emerging Markets

Strictly speaking, these factors are applicable only to sovereign and sovereign agency issuers, based on the inclusion criteria for, say, EMBIG if we happen to use this particular index to estimate emerging markets spread factors. However, most obligors domiciled in these markets carry a risk at least as great as the corresponding sovereign issuer, so that it is reasonable to map the higher grade corporate issuers to the same factor.
Emerging market spread volatilities are shown in Exhibit 8. The spread risk of Latin American and African obligors tends to be above average, with Argentina leading the list with spread risk comparable to a B to CCC-rated corporate. The risk of Asian issuers is, on the other hand, below average and of the same magnitude as interest rate risk. We clearly observe a rich spectrum of risk characteristics that confirms the need to build a separate factor for each market.

**Exhibit 8 — Emerging Market Spread Volatilities as of December 31, 2004**

Empirical Credit Risk

The foregoing discussion has focused largely on securities with relatively low credit risk: so-called investment-grade bonds. Their returns are explained by changes in the term structure and comparatively small spread changes indeed above and below investment-grade bonds are empirically distinguished by the fact that interest rate risk is the largest source of risk for the former, while credit risk often becomes the largest source for the latter.

The basis for the risk models discussed earlier is, effectively, the attribution of bond excess return to interest rates and common spreads as

\[
r_B^t = r_{GOV}^t + (-D_s)\Delta s_B^t + \epsilon_B^t
\]

This equation explains a bond’s return in terms of the return to an equivalent (in the sense of interest rate exposures) government bond, \( r_{GOV}^t \), a market spread factor return \( \Delta s_B^t \), with exposure \(-D_s\) (the negative of the spread duration), and a residual \( \epsilon_B^t \). This model does quite well at explaining returns of high quality bonds, with cross-sectional \( R^2 \) as high as 80% (meaning...
that 20% or less of the cross-sectional variance of bond returns is unexplained). But as we move down the credit quality spectrum, the performance of this return model decreases steadily. By the time we get to the low-quality end of the high yield universe, the fraction of overall bond return variance explained by equation drops below 20%. In other words, as a group, the returns of CCC-rated bonds are substantially explained neither by interest rate movements nor by sector and rating based spread changes. Exhibit 9 shows this systematic decline in model explanatory power (data points labeled “Equation X.8”).

Exhibit 9 — Fraction of Bond Return Variance Explained by Equations (X.8), (X.9), and (X.10) Grouped by Whole Credit Rating (Average of S&P and Moody’s) for US Dollar Bonds, 1996-2003

High yield bond returns are not, as this might seem to imply, mostly issue specific. They are, however, mostly issuer specific. Not surprisingly, returns of bonds of lower credit quality are strongly linked to returns on the issuer’s equity. This is certainly what we expect based on quantitative models of capital structure dating back to Merton, where equity is viewed as a call option on the firm’s assets with a strike price equal to the firm’s liabilities, and a bond is a combination of riskless debt and a short put option on the firm’s assets. The Merton model, based on a highly simplified firm capital structure, makes specific predictions for the relation between bond return, interest rate changes and equity return. The model is too simplified for general application, but it is qualitatively correct: as creditworthiness decreases (default risk increases), a firm’s bonds’ returns become more correlated with its equity returns, and less correlated with returns of default-free bonds.

We can use the market’s assessment of bond credit quality, as measured by its spread over the benchmark, as the input to an empirical model of return linkage. We replace equation (X.8) by an alternative relationship linking the return of a corporate bond to the benchmark return and to the issuer’s equity:

\[ r_i^t = \beta_{IR} r_{GOV}^t + \beta_{E} r_{E}^t + \epsilon_i^t \]  

(X.9)

---

Where $r_E^t$ is the equity return and $\beta_{IR}$ and $\beta_E$ are exposures. Note that the residual $\varepsilon_E^t$ will not have the same value when estimating equation (X.9) as in the context of equation (X.8).

Comparing the two equations, there are two key differences: (1) the bond’s exposure to the benchmark return $r_{GOV}$ is now scaled by the exposure $\beta_{IR}$, which we expect to be close to 1 for high quality bonds, and to decrease with decreasing credit quality and (2) the common-factor spread return, which is not issuer-specific, has been replaced by exposure to the issuer’s equity return, $\beta_E$.

Equation (X.8) serves as the basis for estimating the common factor spread changes $\Delta s_B^t$ — that is, they are not exogenously specified. By contrast, the explanatory returns in equation (X.9), $r_{GOV}^t$ and $r_E^t$ are both exogenous to the model that is, they are determined independently of the bond returns $r_B^t$. Equation (X.8) serves as the basis for estimating the exposures $\beta_{IR}$ and $\beta_E$. The market perception of an issuer’s credit quality can be gauged by observing the spreads on the issuer’s bonds, so we expect to find that $\beta_{IR}$ and $\beta_E$ depend on that spread. A detailed study reveals that $\beta_{IR}$ is only a function of the bond spread and not of the other bond or equity attributes (as far as we have been able to determine). $\beta_E$ depends also on the bond duration (higher for longer duration, not surprisingly), and we have preliminary, unpublished evidence of dependence on firm capitalization and the market liquidity of the firm’s equity.

For risk prediction purposes, we build a heuristic model of $\beta_E$. A simplified version of this model is shown graphically in Exhibit 10. As the curve labeled “Equation (X.9)” in Exhibit 9 indicates, this model performs significantly better for low-quality debt than does equation (X.8). This is not the case for high-grade bonds, however, for which endogenously determined market spreads evidently provide significant explanatory power. We can gain the benefits of both models by fitting market spread changes to the residuals of equation (X.9). The return attribution equation becomes

$$r_E^t = \beta_{IR} r_{GOV}^t + \beta_E r_E^t + (-D_s) \Delta s_B^t + \eta_B^t$$  \hspace{1cm} (X.10)

where $\eta_B^t$ is the remaining residual return. The resulting model has $R^2$s as shown in the upper curve of Exhibit 9, performing similarly to the “rates + spreads” model for high grade credits, and outperforming both models for weaker credits.

---

Exhibit 10 — Spread Dependence of $\beta_{IR}$ and $\beta_{E}$ for US, UK and Euro Domestic Corporate Issues, 1996-2003

**Note:** Data points are based on OLS regression on data binned by spread (OAS). Error bars are based on bootstrap analysis. Curves are non-linear least squares fits of heuristic functional forms to the aggregate data.

For risk forecasting purposes, we therefore replace the interest rate exposures of our original model, such as the shift, twist and butterfly exposures of equation (X.10), with exposures scaled by the factor $\beta_{IR}$, bond by bond, and add equity market exposures, scaled by the factor $\beta_{E}$. For example, a Ford Motor bond with a duration of 5 years and a spread of 180 bp over the government curve, has an estimated $\beta_{IR}$ of 0.81 and a $\beta_{E}$ of 0.036. So the bond’s interest rate exposures are reduced from their “naive” values by approximately 19% and it has a small but nonzero exposure to Ford equity. As shown in the top panel of Exhibit 11, the empirical credit risk model implies relatively small changes in risk exposures for investment grade portfolios, but a significant decrease in interest rate exposure, and non-trivial equity market exposure for high yield portfolios (Exhibit 11, bottom panel). Although the equity market exposure is represented in these figures simply as exposure to a single market factor (equivalent to standard equity $\beta$), in practice we drill down to the multiple factor exposures of the equity risk model implied by the exposures to individual firms.

**Implied Prepayment Risk**

Some markets, in particular the US, Denmark and Japan, have securitized mortgages (MBS) that are largely free of credit risk. As with other fixed-income securities, interest rate movements affect MBS values through changing discount factors. In addition, because of the borrowers’ prepayment options in the underlying loans, MBS have characteristics similar to those of callable bonds. Unlike callable bonds, however, for which the issuers’ refinancing strategies are assumed to be close to optimal, mortgage borrowers may be slow to refinance when it would be financially favorable and to prepay (possibly for non-economic reasons) when it is financially unfavorable. For valuation, this behavior is generally modeled through a prepayment model, giving the
projected paydown rate on an MBS as a function of the security’s characteristics and the current and past economic state. The need for a prepayment model introduces a new source of potential market risk for MBS investors, attributable to model misspecification, changing expectations, or changing market price of risk for exposure to prepayment uncertainty independent of interest rate risk.

One method for accounting for the valuation impact of prepayment risk and uncertainty is through the use of an implied prepayment model, 9 which uses observed market valuations to infer the market price of prepayment risk or, equivalently, to adjust modeled prepayment rates according to market expectations. The calibration is designed to equalize OASs within a universe of MBS’s chosen to broadly sample the range of prepayment exposures.

---

Exhibit 11 — Empirical Duration Adjustment and Equity Market Exposure

Adjusted Portfolio Duration and Equity Market for the Corporate Component of the Investment-Grade Lehman Aggregate US Bond Index in March 2003 and June 2004

**Note:** The effect of bond market spread tightening is visible in the substantial decrease in equity exposure over the 15-month interval between the comparison dates.

**Note:** The empirical credit risk model has a much more dramatic effect on the factor exposures for this portfolio than for the investment grade Lehman portfolio.

The actual implementation of an implied prepayment model in the context of a multifactor risk model simply consists of adding one or more factors to capture changes in market prepayment expectations or (equivalently, for pricing) the market price of prepayment risk. In the simplest case we add just one factor for each major program type. The returns to this factor are obtained as follows. First, we obtain OAS returns for an expanded universe of MBS within the same program, including to-be-announced issues (TBAs) and more seasoned generics. Then, by
regression, we obtain returns for a spread factor and a prepayment factor. Exposures to the spread factor are spread durations. Exposures to the prepayment factor are determined by shocking the overall speed of the prepayment model and observing percentage sensitivity of the valuation model. The sign convention is such that premiums generally have positive prepayment exposure, and discounts negative. Typical prepayment exposures range from −0.01 to 0.06. For example, if an MBS with prepayment exposure of 0.05 were revalued subject to a 10% increase in prepayment speeds, its price would drop by about half a percent (0.05 times 0.1). Prepayment factor volatilities are usually on the order of one, yielding a prepayment risk that can be as large as 6% for highly exposed securities. (To date, we have implemented this model only for the US market.)

The addition of refinancing factors results in a distinct improvement in the explanatory power of the factor model. For example, from 1996 through 2003, a model that includes a prepayment factor captures on average 18% more of the variance of the monthly returns of conventional 30-year issues than a spread-only model. In some months with small spread returns, the prepayment factor by itself accounts for 50% or more of the observed returns variation.

Implied Volatility Risk

The value of instruments with no embedded options is only a function of the current term structure. In contrast, the uncertain character of future interest rates has a significant impact on the analysis of instruments with optionality, for example callable bonds, mortgage pass-throughs, or explicit options like caps and swaptions. Such instruments are exposed to the market’s varying expectation of the volatility of the term structure, or equivalently, to variations in the market price of risk for interest rate volatility. The basic idea underlying a simple implied volatility risk model is to calibrate a stochastic interest rate model to match observed market prices of interest rate options. The variation over time of the calibration constants then gives rise to implied risk factors.

Consider for instance a Mean-Reverting Gaussian (MRG) model, also called the Hull-White model, which assumes that the increment to the short rate \( \frac{dr}{r} \) is a normal random variable with reversion to a long-term mean. In this model, there is a closed form for the price of European swaptions, and given a term structure, we can adjust the model parameters to fit the market prices of swaptions that have a LIBOR at various tenors and expiries. Now fully specified, the MRG model determines the volatility of all forward rates, spot rates and yields.

Exposure of a security to the implied volatility factor is then determined by numerical differentiation, analogously to duration. In MSCI Barra’s model implementation, the factor is the logarithm of the 10-year yield, which has the advantage of capturing volatility of the portion of the term structure relevant for most optionable bonds and MBSs. If we denote the factor by \( V \), the exposure of a security is then the percentage change in price per unit increase of \( V \), that is, per percentage change in 10-year volatility.

---

10 As measured by the coefficient of determination \( R^2 \) of the models for a common universe of mortgages
In practice, one computes this derivative as follows:

\[ D_{\text{V}} = \frac{1}{P} \frac{\partial P}{\partial V} = \frac{1}{P} \frac{\partial P}{\partial \sigma} \left( \frac{\partial V}{\partial \sigma} \right)^{-1} \]  

(X.11)

where \( P \) is the security price and \( \sigma \) is the volatility of the short rate.

For MBSs and callable bonds, this exposure is typically negative, because they contain embedded short option positions. Increased volatility (positive factor return) increases the value of the implicit short call position, resulting in a negative asset return. For these securities, exposures generally range between 0 and \(-0.06\). In the major markets, the volatility of \( V \) is usually on the order \(0.2 \text{ yr}^{-1}\) yielding an implied volatility risk that can reach \(1\%\) for bond with at-the-money embedded options.

**Specific Risk**

Specific return is residual return not explained by common factors. For securities without significant default risk, this is generally viewed as some form of asset-level basis risk. For bonds e.g., domestic government bonds, this can be straightforwardly forecasted from the standard deviations of the residuals. (More careful modeling would account for liquidity effects such as those affecting benchmark bonds.)

Bonds bearing significant default risk have a firm-specific contribution to their risk. The size of this risk can be forecast reasonably well with a reduced form model based on credit migration probabilities. Historical credit migration rates are reported by the major rating agencies, and can be used to estimate future probabilities. Given these probabilities, the specific return variance of the bonds from an issuer can be written as

\[ \sigma_{\text{spec}}^2 = \sum_j p_{i \rightarrow j} \left[ D(s_j - s_i) - r_m \right]^2 + p_{i \rightarrow d} (1 - R - r_m)^2 \]  

(X.12)

where

- \( p_{i \rightarrow j} \) = the one-period probability of transitioning from rating \( i \) to \( j \)
- \( p_{i \rightarrow d} \) = the one-period default probability
- \( D \) = the bond’s spread duration
- \( s_i \) = the average spread level observed amongst bonds with rating \( i \)
- \( R \) = the recovery as a fraction of market value (for which we use a standard 50\% estimate)

\[ r_m = \left[ \sum_j p_{i \rightarrow j} D(s_i - s_j) \right] + p_{i \rightarrow d} (1 - R) \]  

\( s \) the mean expected return

The main premise of this model is that the variance of issuer-specific bond returns arises from credit events. Empirically, credit events of sufficient magnitude to cause one or two whole-step rating changes make the largest contribution to the variance. Note that we are not concerned in this formula with the lag between credit events and agency rating changes. We care only about the average rate of such events, and as long as agency ratings eventually reflect credit quality changes, the reported transition probabilities give good estimates for the rate of these events.
Outside of the US market, there is neither sufficient breadth of credit quality nor sufficient history to reliably estimate the small probabilities of large credit events. However, the main rating firms combine non-US and US data to give global credit migration rates based on historical experience. We use these reported global estimates of \( p_{i \rightarrow j} \) and \( p_{i \rightarrow d} \) as the basis for the credit specific risk model.

The model also requires average spread levels observed within each rating category. Since these levels are market-dependent, so are the credit event risk forecasts. A consequence is that this approach can only be implemented in highly liquid markets, where there are enough corporate bonds to robustly estimate average spread levels in practice, markets for which we can construct sector-by-rating credit factors.

Given the transition probabilities and spread levels for the different rating classes, the model estimates the distribution of issuer-specific bond returns in a linear approximation from the spread differences and the bond duration. The return variance is then computed from the discrete distribution.

In markets where there is not enough data to construct this detailed model (e.g., because there aren’t enough corporate bonds with reported prices across the full range of ratings), the simplest solution is a linear model of residual spread volatility, increasing as a function of spread level:

\[
\sigma_{\text{spec}} = (a + b \cdot s)D
\]

where \( s \) is the bond’s spread and \( D \) is the bond’s duration. The two constants \( a \) and \( b \) are fitted in each market using observed residual returns.

**Currency Risk**

Un-hedged currency risk is a potentially large source of risk for global investors. In addition to being a large source of return volatility, currency risk can be highly variable in time. We therefore need a model capable of quickly adjusting to new risk regimes and responsive to new data. Various forms of General Auto-Regressive Conditional Heteroskedastic (GARCH) models have been used to this effect. Such models express current return volatility as a function of previous returns and forecasts. For instance, the GARCH(1,1) model takes the form:

\[
\sigma_t^2 = \omega^2 + \beta(\sigma_{t-1}^2 - \omega^2) + \gamma(r_{t-1} - \omega^2)
\]

where:

- \( \sigma_t^2 \) = conditional variance forecast at time \( t \)
- \( \omega^2 \) = unconditional variance forecast
- \( \beta \) = persistence
- \( \gamma \) = sensitivity
- \( r_{t-1} \) = observed return from \( t-1 \) to \( t \)

The constants \( \beta \) and \( \gamma \) are required to be non-negative, and in order to avoid runaway behavior, the condition \( \beta + \gamma \leq 1 \) also must hold. The larger the sensitivity \( \gamma \), the more responsive the model is to a new large return. Conversely, larger values of the persistence \( \beta \) imply more weight given to a longer history.
Using daily exchange rates insures the convergence of GARCH parameters and minimizes the noise in forecasts based on a short history. To get a monthly risk forecast $\sigma_{t,n}$ from the one-day forecast of equation (X.14), we use the scaling formula (which follows from iteration of equation (X.14)):

$$\sigma_{t,n}^2 = n\omega^2 + \frac{1 - (\beta + \gamma)^n}{1 - (\beta + \gamma)} (\sigma_t^2 + \omega^2)$$  \hspace{1cm} (X.15)

where $n$ is the number of business days in a month, typically 20 or 21.

Exhibit 12 shows US dollar versus Euro returns from 1994 to 2000. Note how volatility forecasts (gray lines) quickly adjust to periods of small or large returns. The overall currency risk is large compared to interest rate risk. From the perspective of a US investor, a German government bond with a 5-year duration has annualized interest rate risk of about 350 bp and currency risk of about 800 bp. The volatilities of several other currencies from a US dollar perspective are plotted in Exhibit 13, and typically range from roughly 6.5% to 10% per year.

**Exhibit 12 — US Dollar Against Euro Currency Returns and Volatility**

*Note:* The Euro is proxied by the Deutschmark prior to 1999.

**Global Model Integration**

Common factors, returns, exposures, and a specific risk model: everything is there except for one last critical ingredient: the covariance matrix. Building a sensible covariance matrix for more than a few factors is a complicated task that involves solving several problems.
Coping with Incomplete Return Series

Factor return series also often have different lengths, with some series starting earlier than others. Return series can also have gaps. A consequence is that the matrix whose elements are given by the standard pair-wise formula for the covariance of two series will not, in general, be positive semi-definite, and is therefore not a covariance matrix. A standard estimation technique in this situation is the EM algorithm.\(^{12}\)

The product of this algorithm is a maximum likelihood estimator for the covariance matrix of the observed incomplete data.

Global Integration

Barra’s latest global fixed income model includes nearly 500 factors, yielding over 120,000 covariances. (This does not include the covariances of fixed income factors with equity factors, which are relevant for modeling high-yield bonds as described earlier.) In many cases, the factor returns series include no more than 30 to 40 periods. With such a small sample size compared to the number of factors, we have a severely under-determined problem and are virtually assured that the covariance forecasts will show a large degree of spurious linear dependence among the factors. One consequence is that it becomes possible to create portfolios with artificially low risk forecasts (for example, by use of an optimizer). The structure of these portfolios would be peculiar, e.g., they might be overweight Japanese banks, apparently hedged by an underweight in euro Industrial and Telecom.

Reducing the number of factors would compromise the accuracy of our risk analysis at the local level. However, we have seen for instance that the higher grade developed credit markets are

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largely independent so that we do not need all cross-currency covariances to describe the
coupling between these two markets. Using our knowledge of the market in a more systematic
fashion could go a long way in reducing the spurious correlations amongst factors.

The structured approach presented in Stefek et al.\textsuperscript{13} is one solution to this problem. In this
method, factor returns are decomposed into a global component and a purely local component,
extactly as we already decomposed asset returns into systematic and non-systematic returns.

For instance, in the US dollar market we can write:

$$f_{USD} = X_{USD}^T \cdot g_{USD} + \epsilon_{USD}$$  \hspace{1cm} (X.16)

where

- $f_{USD}$ = the vector of factor returns for the US market
- $g_{USD}$ = the vector of global factor returns for the US market
- $X_{USD}$ = the exposure matrix of the US local factors to the US global factors
- $\epsilon_{USD}$ = the vector of residual factor returns not explained by global factors, or purely local
  returns

Equation (X.16) can be easily extended to all the original factors in the model. Since purely local
returns are now by construction uncorrelated across markets and also uncorrelated with global
returns, the covariance matrix can then be written as:

$$F = XG^T X + \Lambda$$  \hspace{1cm} (X.17)

where

- $G$ = the covariance matrix of global factors
- $X$ = the exposure matrix of the local factor to the global factors
- $\Lambda$ = the covariance matrix of local factors

The choice of global factors is based on econometric considerations. For instance, there is a
strong link between interest rates across currencies, especially for the major markets. Given that
interest rate risk is a critical component of fixed income risk, we want to insure that correlations
between interest rate factors in different markets are modeled as accurately as possible. This can
be achieved by making the Shifts, Twists and Butterflies, and implied volatility factors global. We
also found that to a large degree, credit factors behave independently of factors in other markets.
As a result, we know that we will gain very little by choosing more than a few global credit factors
in each developed market. However, the link between major credit markets appears to become
stronger as credit quality decreases. In choosing global factors, we also want enough granularity
to capture such nuances. One possible approach is to create two global average investment-
grade spread factors as well as an average high-yield spread factor in markets where there is a
reasonably developed speculative debt.

Global factors could typically include:

- The Shift, Twist, Butterfly and implied volatility factors
- The swap spread factors
- AAA/AA and A/BBB average high-grade credit spread factors in the euro zone, US, UK, Japan, Canada and Switzerland
- Average high-yield credit spread factors in the euro zone, US, UK and Japan
- An average emerging market spread

Unfortunately, we cannot stop there and use equation (X.17). The benefit of using global factors is that they help compute cross-market terms and constitute the skeleton of the matrix. The drawback is a loss of resolution at the local level. A solution to this problem is to replace local blocks by a local covariance matrix computed using the EM algorithm and the full set of original local factors, or “scale” local covariance blocks.\(^{14}\)

At this point, we have a method for building a model that reconciles two conflicting goals, that is, provide a wide coverage of markets and securities while permitting an accurate and insightful analysis, particularly at the local level.

**The Model in Action**

The risk characteristics of several typical indices are presented in Exhibit 14. We find again that currency risk dominates by far local risk. US investors holding an un-hedged portfolio of yen denominated bonds incur a currency risk that is about four times larger than the interest rate risk. For investment-grade portfolios, interest rate risk represents most of the local risk. It is only for high-yield and emerging market portfolios that spread risk contributes a significant portion of local risk. In fact, for an index such as JP Morgan EMBIG, spread is about the same as interest rate risk. Local risk is the smallest in the yen market and the largest in the US dollar market owing to the relatively large US interest rate factor volatilities (see Exhibit 2).

For diversified portfolios in which bonds with embedded options (including mortgages) represent only a small fraction of the total value, there is very little volatility and prepayment risk. However, such risks can become more significant in portfolios of mortgages and can even exceed spread risk, as in the US mortgage index as shown in the Exhibit.

**Summary**

Although the models and methodologies that we have described in this paper are for the most part relatively standard, two are more recent additions to the realm of risk management. The first is the inclusion of equity exposure in the modeling of fixed income risk. The second is a structural method to aggregate single market models into a global risk model.

Adequately measuring risk requires sophisticated methods and considerable care. A good risk model should, at a minimum, provide a broad coverage without sacrificing accuracy, retain details but remain parsimonious and be responsive to market changes. The sources of risk are many, and their respective importance depends on the asset. Certainly, there is no shortage of challenges.

The authors thank Avaneesh Krishnamoorthy for assistance with the risk computation method to aggregate single market models into a global risk model.

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